Modeling a heterogeneous network with TCP connections using fluid flow approximation and queuing theory

Hiroyuki Hisamatsu[a] and Hiroyuki Ohsaki[a] and Masayuki Murata[b]

[a] Graduate School of Information Science and Technology, Osaka University, 1-32, Machikaneyama-cho,Toyonaka, Osaka, Japan

[b] Cybermedia Center, Osaka University, Osaka, Japan

ABSTRACT

In the current Internet, most of the traffic is transmitted by TCP (Transmission Control Protocol). In our previous work,[1] we have proposed a modeling approach for the entire network, including TCP congestion control mechanisms operating at source hosts and the network seen by TCP connections, as a single feedback system. However, our analytic model is limited to a simple network, where TCP connections have the identical propagation delay. In this paper, we therefore extend our analytic approach to a more generic network, where multiple TCP connections are allowed to have different propagation delays. We derive the packet loss probability in the network, the throughput and the average round-trip time of each TCP connection in steady state. By presenting several numerical examples, we quantitatively investigate how the fairness among TCP connections is degraded when multiple TCP connections with different propagation delays share the single bottleneck link.

Keywords: T CP (Transmission Control Protocol), Propagation Delay, Steady State Analysis, Fairness

1. INTRODUCTION

The Internet uses a window-based flow control mechanism in TCP (Transmission Control Protocol), which is a sort of feedback based congestion control mechanisms.[2] TCP has two fundamental mechanisms: a packet retransmission mechanism and a congestion avoidance mechanism. The packet retransmission mechanism re-sends lost packets in the network for realizing reliable data transfer between source and destination hosts. The congestion avoidance mechanism controls the packet emission process from a source host according to the congestion status for utilizing network resources effectively.

The great portion of the traffic in the Internet is transmitted by TCP.[3, 4] Since the algorithm of TCP is complicated, the simulation technique has been widely used to evaluate the performance of TCP. However, the performance evaluation of TCP using mathematical analysis such as fluid approximation or queuing theory has been performed actively in recent years.[1, 5-11] For instance, by assuming that a packet loss probability in the network and the round-trip time of TCP are known, the TCP throughput in steady state is derived.[5, 6] However, the packet loss probability in the network and the round-trip time of TCP are not constant since they are changed by the congestion control of TCP. Therefore, performance analyses of TCP that explicitly model interaction between the congestion control mechanism of TCP and the network have been performed.[1, 7-10]

Those analyses that explicitly model interaction between the congestion control mechanism of TCP and the network can be classified into two categories according to the modeling approach of the bottleneck link in the network. First, by assuming the bottleneck router to be a conventional Drop-Tail router, the bottleneck link in the network is modeled by a queueing system such as $M/M/1/m$ or $M/D/1/m$.[1, 7, 8] On the other hand, by assuming the bottleneck router to be an AQM (Active Queue Management) router, the bottleneck link is modeled using fluid flow approximation.[9, 10] Currently,

Further author information: (Send correspondence to Hisamatsu)
H.H.: E-mail: hisamatu@anarg.jp, Telephone: +81 06 6850 6863
O.H.: E-mail: oosaki@ist.osaka-u.ac.jp, Telephone: +81 06 6850 6614
M.M.: E-mail: murata@cmc.osaka-u.ac.jp, Telephone: +81 06 6850 6860

most routers operating in the current Internet are Drop-Tail routers. Therefore, the former analytic approach of modeling the bottleneck link using the queuing theory should have larger application area than the later one.

In our previous work,[1] we have proposed an approach for modeling the network including TCP running at a source host as a single feedback system. Our approach was to separately model the TCP congestion control mechanism and the network using fluid flow approximation and queuing theory, respectively. We have modeled the congestion control mechanism of TCP as a SISO (Single Input and Single Output) system, where the input to the system is an observed packet loss probability in the network and the output is a TCP window size. We have also modeled the network seen by TCP as another SISO system, where the input to the system is the TCP window size and the output is the packet loss probability in the network. By combining these SISO systems, we have modeled the network including the congestion control mechanism of TCP running on a source host as a single feedback system.

Note that, in our previous work,[1] we have analyzed a simple network where the propagation delay of each TCP connection is identical. However, in real networks, each TCP connection should have different propagation delays. On the other hand, it is known from simulation studies[12] and from analytical studies[5, 6, 11, 13] that the TCP throughput is dependent on its round-trip time. In general, the larger the propagation delay of a TCP connection is, the larger its round-trip time is. Thus, when multiple TCP connections with different propagation delays share the single bottleneck link, it is expected that fairness among TCP connections is deteriorated.

For instance, if the packet loss probability in the network is constant, it is known that the TCP throughput in steady state is inverse proportional to its round-trip time.[5, 6, 11, 13] Note that, in those analytical studies, the packet loss probability in the network and the round-trip time of TCP connections are assumed to be known. However, in reality, neither the packet loss probability in the network nor the round-trip time of TCP is constant, and both are changed by the congestion control mechanism of TCP. Thus, for analyzing multiple TCP connections with different propagation delays, an analytic approach that explicitly models dynamics of the packet loss probability in the network and the round-trip time of TCP is required.

In this paper, by extending our previous modeling approach,[1] we model both TCP and the network where the packet loss probability in the network and the round-trip time of TCP are dynamic. We analyze a more general network where each TCP connection has a different propagation delay. The modeling approach proposed in this paper is different from our previous one[1] in the following two points. First, to improve the accuracy of approximate analysis, we use $M/D/1/m$ queuing system instead of $M/M/1/m$ queuing system as a bottleneck link model. Second, to analyze the steady state behavior of TCP, we model the congestion control mechanism of TCP using the TCP throughput equation[5] instead of fluid flow approximation.

We first present the steady state analysis for a network with a single TCP connection, and derive the throughput and the round-trip time of TCP, and the packet loss probability in the network. We then extend our steady state analysis to a network where each TCP connection is allowed to have a different propagation delay. We derive the throughput and the round-trip time of TCP, and the packet loss probability in the network. By showing several numerical examples, we quantitatively investigate how the fairness among TCP connections is degraded when multiple TCP connections with different propagation delays share the single bottleneck link. We show that the fairness among TCP connections becomes worse as the packet processing delay in the router's buffer becomes smaller (i.e., larger processing speed of the bottleneck router and/or smaller buffer capacity of the bottleneck router).

In references,[7, 8] the steady state behavior of TCP is analyzed by modeling the congestion control mechanism of TCP using either Markov chain or queueing theory. Similar to the analytic approach proposed in this paper, those analyses explicitly model interaction between the congestion control mechanism of TCP and the network where each TCP connection is allowed to have a different propagation delay. However, our analytic approach is superior to those approaches in the following points. First, our analytic approach can be extended to analyze the transient state behavior of TCP.[1] Second, our analytic approach requires less computational burden than those approaches based on Markov chain or queueing theory since our analytic approach uses the TCP throughput equation.[5]

Organization of this paper is as follows. In Section 2, we explain our analytic model and define several symbols. In Section 3, we model the network including the congestion control mechanism of TCP running at the source host as a single feedback system. We then derive the throughput and the round-trip time of TCP, and the packet loss probability in the

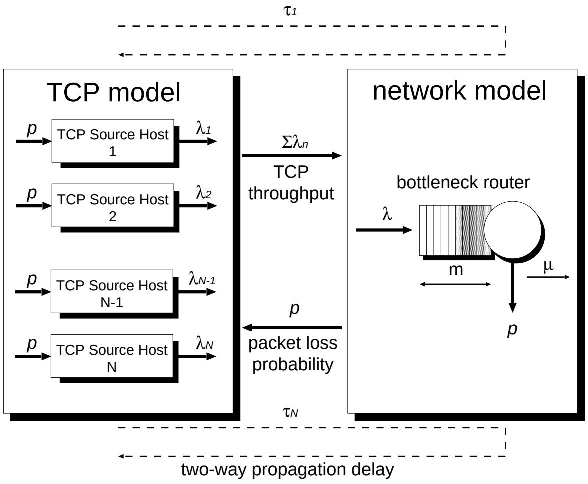


Figure 1: Analytic model.

Table 1. Definition of symbols.

N	the number of TCP connections
μ	packet processing speed of bottleneck router
m	buffer size of bottleneck router
τ_n	two-way propagation delay of n_th TCP connection
r_n	round-trip time of n_{th} TCP connection
λ_n	throughput of n_{th} TCP connection
p	packet loss probability in the network

network. In Section 4, by showing several numerical examples, we quantitatively investigate how the fairness among TCP connections is degraded when multiple TCP connections with different propagation delays share the single bottleneck link. Finally, in Section 5, we conclude this paper and discuss future works.

2. ANALYTIC MODEL

In this paper, by extending our previous modeling approach,[1] we analyze the network with multiple TCP connections where each TCP connection is allowed to have a different propagation delay. Figure 1 illustrates our analytic model. The number N of TCP connections share the single bottleneck router (i.e., the router connected to the bottleneck link), and each TCP connection has a different propagation delay. We assume that the number of TCP connections is fixed. The packet sending rate of n_{th} ($1 \leq n \leq N$) TCP connection is denoted by λ_n, and its propagation delay is denoted by τ_n. TCP uses a window-based flow control mechanism that controls the number of packets that the source host can inject into the network during its round-trip time. However, in this paper, we use the instantaneous packet sending rate ($\simeq$ window size / round-trip time). We assume that "propagation delay" includes any delays except the queueing delay and the processing delay at the bottleneck router. We also assume that the "propagation delay" is constant. Table 1 summarizes the definition of symbols used throughout this paper.

The congestion control mechanism of TCP changes its window size according to occurrence of packet losses in the network. Hence, there exists a tendency that when the packet loss probability is small, the window size is large (i.e, the sending rate is large). On the contrary, when the packet loss probability is large, the window size tends to be small (i.e, the sending rate is small). We therefore model the congestion control mechanism of TCP at the source hosts as N systems where the input to the system is the packet loss probability in the network and the output from the system is the TCP throughput (see Fig. 1).

On the other hand, the network seen by TCP behaves such that when the number of packets entering the network increases, some packets are awaited at router's buffer. This sometimes causes buffer overflow, resulting in a packet loss. So the packet loss probability becomes large when the number of packets entering the network increases. Thus, the network seen by TCP can be modeled as a system, where the input to the system is the TCP throughput and the output from the system is the packet loss probability (see Fig. 1).

Note that, we make following assumptions in our analysis: (1) to focus on the steady state behavior of TCP, all TCP connections are assumed to operate in the congestion avoidance phase, (2) the bottleneck router is a Drop-Tail router with a single FIFO queue for all TCP connections, (3) the TCP packet size is fixed, (4) all TCP connections have infinite data to transfer, (5) the packet loss only occurs at the bottleneck router due to its buffer overflow, and (6) the maximum window size of TCP is sufficiently larger than the bandwidth–delay product of the network.

3. STEADY STATE ANALYSIS

3.1. Case of a single TCP connection

We first present the steady state analysis for the network with a single TCP connection. The TCP source host is modeled by a SISO (Single Input and Single Output) system; i.e., the packet loss probability p in the network is the input to the system, and the TCP throughput λ is the output. Let p and r be the packet loss probability in the network and the round-trip time of the TCP connection, respectively. The TCP throughput $\lambda(p, r)$ is approximately given by the following equation.[5]

$$\lambda(p, r) \simeq \frac{1}{r}\sqrt{\frac{3}{2\,b\,p}} \tag{1}$$

In the above equation, b (usually $b = 1$ or $b = 2$) is the number of packets required for the TCP destination host to send an ACK packet.

We then model the network seen from the TCP source host by a single FIFO (First-In First-Out) queue. Provided that queueing delay occurs only at the bottleneck router, the bottleneck router is modeled by a M/M/1 queuing system. In other words, the network is modeled by a SISO system, where the input is the TCP throughput λ and the output is the packet loss probability p at the bottleneck router. Let λ and m be the TCP throughput and the buffer size of the bottleneck router. The packet loss probability in the network is approximated as

$$p(\lambda) \simeq 1 - \sum_{k=0}^{m} P_k(\lambda) \tag{2}$$

where $P_k(\lambda)$ is the state probability of a M/M/1 queuing system. By letting μ be the packet processing speed of the bottleneck router, $P_k(\lambda)$ is given by

$$P_k(\lambda) = \left(1 - \frac{\lambda}{\mu}\right)\left(\frac{\lambda}{\mu}\right)^k \tag{3}$$

Thus, $p(\lambda)$ is obtained from Eqs. (2) and (3) as

$$p(\lambda) \simeq \left(\frac{\lambda}{\mu}\right)^{1+m} \tag{4}$$

Since the window-based flow control mechanism of TCP operates around the 100% offered traffic load, $p(\lambda)$ in the above equation can be approximated by

$$p(\lambda) \simeq \frac{\lambda + \lambda\,m - m\,\mu}{\mu} \tag{5}$$

We then derive the round-trip time of the TCP connection. Let τ be the propagation delay of a TCP connection. We assume that the queueing delay occurs only at the bottleneck router. For a given TCP throughput λ, the TCP round-trip time $r(\lambda)$

is obtained using the average queue length $L = \lambda/(\mu - \lambda)$ of a M/M/1 queueing system and the Little's theorem $N = \lambda T$. Thus,

$$r(\lambda) \quad = \quad \frac{L}{\lambda} + \tau = \frac{1}{\mu - \lambda} + \tau \tag{6}$$

Let λ^*, r^*, and p^* be the TCP throughput, the round-trip time of the TCP connection, and the packet loss probability in the network in steady state, respectively. From Eqs. (1), (5), and (6), the following relations are satisfied in steady state.

$$\lambda(p^*, r^*) = \lambda^*$$
$$p(\lambda^*) = p^*$$
$$r(\lambda^*) = r^*$$

By solving these equations, we can obtain the TCP throughput λ^*, the round-trip time r^* of the TCP connection, and the packet loss probability p^* in steady state. For example, the TCP throughput λ^* in steady state is given by the solution of the following equation.

$$\frac{(\mu - \lambda)\sqrt{\frac{3\,\mu}{2\,b\,(\lambda + \lambda\, m - m\,\mu)}}}{1 - \lambda\,\tau + \mu\,\tau} = \lambda \tag{7}$$

3.2. Case of multiple TCP connections

In what follows, we extend the previous analysis to a network with heterogeneous TCP connections. Let N be the number of TCP connections, λ_n be the throughput of n_{th} TCP connection, and r_n be the round-trip time of n_{th} TCP connection. We assume random packet losses in the network. From Eq. (1), for a given packet loss probability p in the network, the throughput of each TCP connection is given by

$$\lambda_n(p, r_n) \quad \simeq \quad \frac{1}{r_n}\sqrt{\frac{3}{2\,b\,p}} \tag{8}$$

We then model the network seen from TCP source hosts as a MISO (Multi Input and Single Output) system, where the inputs are throughputs λ_n of TCP connections and the output is the packet loss probability p in the network. From Eq. (2), the packet loss probability in the network with multiple TCP connections, $p(\lambda_1, \cdots, \lambda_N)$, is obtained as

$$p(\lambda_1, \cdots, \lambda_N) \quad \simeq \quad 1 - \sum_{k=0}^{m} P_k(\sum_{n=1}^{N} \lambda_n) \tag{9}$$

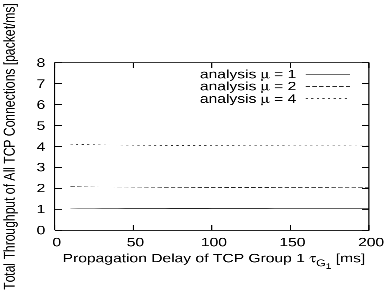
Similarly, for a given TCP throughput λ_n, the round-trip time $r_n(\lambda_n)$ of n_{th} TCP connections is obtained from Eq. (6).

$$r_n(\lambda_n) \quad = \quad \frac{1}{\mu - \sum_{n=1}^{N} \lambda_n} + \tau_n \tag{10}$$

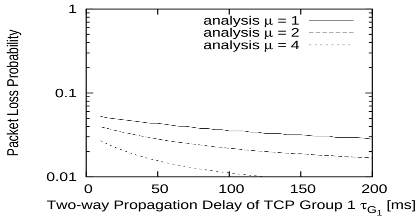
where τ_n is the propagation delay of n_{th} TCP connection.

Finally, we derive the equilibrium in steady state. Let λ_n^*, r_n^*, and p^* be the throughput of n_{th} TCP connection, the round-trip time of n_{th} TCP connection, and the packet loss probability in steady state, respectively. We have the following equations from Eqs. (8), (9), and (10).

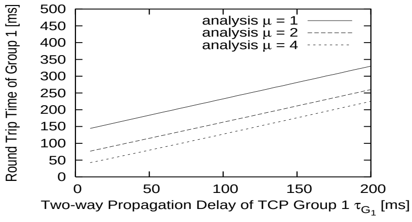
$$\begin{aligned}
\lambda_1(p^*, r_1^*) &= \lambda_1^* \\
&\vdots \\
\lambda_N(p^*, r_N^*) &= \lambda_N^* \\
p(\lambda_1^*, \cdots, \lambda_N^*) &= p^* \\
r(\lambda_1^*) &= r_1^* \\
&\vdots \\
r(\lambda_N^*) &= r_N^*
\end{aligned}$$



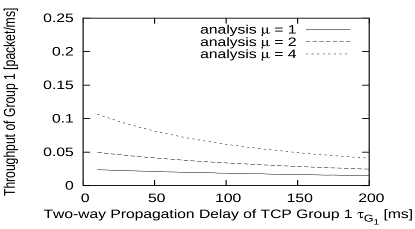
(a) Total throughputs of all the TCP connections



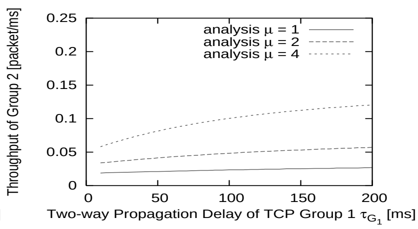
(b) Packet loss probability in the network



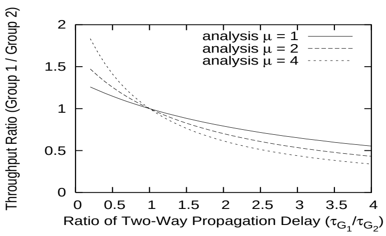
(c) Roud-trip time of TCP connections in G_1



(d) Throughput of TCP connections in G_1



(e) Throughput of TCP connections in G_2



(f) Relation between the propagation delay ratio and the throughput ratio

Figure 2: Numerical examples for different processing speed of the bottleneck router μ ($\tau_{G_2} = 50$ [ms], $m = 150$ [packet], $N = 50$)

By numerically solving these equations for λ_n^*, r_n^*, and p^*, the throughput and the round-trip time of n_{th} TCP connection, and the packet loss probability in the network can be obtained.

4. NUMERICAL EXAMPLES

By presenting several numerical examples, we investigate the steady state behavior of TCP when each TCP connection has a different propagation delay. In numerical examples, the buffer size of the bottleneck router m is 150 [packet], and the processing speed of the bottleneck router μ is either 1, 2, or 4 [packet/ms]. The number of TCP connections N is fixed at 50, and these TCP connections are classified into two groups called G_1 and G_2 according to their propagation delays. Propagation delays of all TCP connections in G_1 are identically denoted by $\tau_{G_1} = \tau_i$ ($1 \leq i \leq 25$) [ms], and propagation delays of TCP connections in G_2 are by $\tau_{G_2} = \tau_i$ ($26 \leq i \leq 50$) [ms].

Figure 2 shows analytic results for different propagation delays of TCP connections in G_1 whereas the propagation delay of TCP connections in G_2 is fixed. This figure shows (a) the total throughputs of all TCP connections ($\equiv \sum_{i=1}^{N} \lambda_i$), (b) the packet loss probability in the network, (c) the round-trip time of TCP connections in G_1, (d) the throughput of TCP connections in G_1, (e) the throughput of TCP connections in G_2, and (f) the relation between the propagation delay ratio (τ_{G_1}/τ_{G_2}) and the throughput ratio. Each figure in Fig. 2 contains analytic results for different processing speeds of the bottleneck router: $\mu = 1$, 2, and 4 [packet/ms].

First, Fig. 2(a) shows that aggregated TCP connections achieve roughly 100 % throughput regardless of the propagation delay of TCP connections. This indicates that TCP can effectively utilize network resources even when propagation delays of TCP connections are different. Next, we focus on the packet loss probability in the network (see Fig. 2(b)). Note that, since we assume the bottleneck router to have a FIFO buffer in our analysis, the packet loss probability observed by each TCP connection is identical. Figure 2(b) shows that the packet loss probability in the network decreases as τ_{G_1} becomes large when propagation delays of TCP connections are different. This phenomenon can be explained by the fact that the congestion control mechanism of TCP tries to fully utilize network resources. Namely, the round-trip time becomes large as the propagation delay becomes large. The larger the round-trip time becomes, the smaller the receiving ACK rate by the source host becomes, resulting in a slower window size increase. Consequently, the packet loss probability in the network decreases. Note that, although the packet loss probability becomes large as the propagation delay becomes large, the total TCP throughput is almost 100% in all cases (see Fig. 2(a)). Hence, in terms of the total TCP throughput, variation in the packet loss probability caused by the difference in propagation delays of TCP connections is not problematic.

On the other hand, one can find that the packet loss probability in the network decreases as the processing speed of the bottleneck router μ becomes large. This can also be explained by the fact that the congestion control mechanism of TCP tries to fully utilize network resources. Namely, if the buffer size is fixed, the round-trip time observed by each TCP connection is almost constant regardless of the processing speed of the bottleneck router (see Fig. 2(c)). Thus, when the processing speed of the bottleneck router is large, TCP must enlarge its window size to utilize network resources, resulting in a small packet loss probability in the network.

Furthermore, we focus on the round-trip time of TCP connections in G_1 (see Fig. 2(c)). TCP is a congestion control mechanism based on a packet loss. Moreover the bottleneck router is a Drop-Tail router in our analysis. For this reason, the average number of packets in the buffer takes a value close to the router's buffer size. For instance, when $\tau_{G_1} = 50$ [ms] and $\mu = 1$ [packet/ms], the average number of packets in the buffer is 128.50 [packet], which is close to the router's buffer size. Furthermore, one can find that the round-trip time of TCP connections becomes small as the processing speed μ of the bottleneck router increases. This can be explained as follows. First, the round-trip time of a TCP connection is given by the sum of its two-way propagation delay and the packet waiting time in the router's buffer (see Eq. (10)). The packet waiting time in the router's buffer takes a small value as the processing speed μ of the bottleneck router increases. Consequently, the round-trip time of a TCP connection becomes small as the processing speed μ of the bottleneck router increases.

As shown in Fig. 2(a), aggregated TCP connections achieve roughly 100 % throughput regardless of the propagation delay of TCP connections. However, there is a serious problem from a viewpoint of fairness among TCP connections. As shown in Figs. 2(d) and 2(e), when propagation delays of all TCP connections are identical (i.e., $\tau_{G_1} = \tau_{G_2}$), each TCP throughput is also identical, resulting in a fair share of the bottleneck link bandwidth. However, when propagation delays of TCP connections are different, TCP connections with smaller propagation delays gain higher throughput than ones with larger propagation delays. For instance, when the ratio of propagation delays is 1/2 ($2\tau_{G_1} = \tau_{G_2}$), the throughput of TCP connections in G_1 is 0.16 [Mbps], and the throughput of TCP connections in G_2 is 0.48 [Mbps]. So, the ratio of TCP throughputs is about 1/3. This is because TCP increases its window size every receipt of an ACK packet in its congestion avoidance phase. Namely, the packet loss probability observed by all TCP connections are identical since all TCP connections share the single bottleneck link. However, TCP connections with smaller round-trip times receive ACK packets more frequently than ones with larger round-trip times. TCP connections with smaller round-trip time increase their window sizes more quickly, so that they can achieve higher throughput than ones with larger round-trip times. In addition, some AQM routers that change their packet marking probability according to TCP connection's throughput is proposed for improving unfairness among TCP connections.[14-16] For instance, an AQM router realizing better fairness among TCP connections might be designed by utilizing our analytic result.

Figure 2(f) is generated from Figs. 2(d) and 2(e) for displaying the relation between the propagation delay ratio (τ_{G_1}/τ_{G_2}) and the throughput ratio. This figure shows that the slope of the throughput ratio is large when the propagation delay ratio is around 1.0. On the other hand, when the propagation delay ratio is large (e.g., around 3.0), the slope of the throughput ratio is small. This indicates that competing TCP connections receive greatly different throughputs by the slight difference in their propagation delays, resulting in unfairness among TCP connections. This is a serious fairness issue of TCP. Furthermore, one can find that the throughput ratio becomes large as the processing speed μ of the bottleneck router becomes large. This means that unfairness among TCP connections will get worse as the network capacity becomes larger in the future. Such tendency can be explained as follows. The waiting time in the router becomes small as the processing speed of the bottleneck router becomes large (see Fig. 2 (c)). Consequently, the difference in propagation delays of TCP connections directly affects the difference in their round-trip times.

5. CONCLUSION AND FUTURE WORKS

In this paper, we have analyzed the steady state behavior of TCP when each TCP connection has a different propagation delay. We have derived the throughput and the round-trip time of TCP, and the packet loss probability in the network. By presenting numerical examples, we have quantitatively investigated how the fairness among TCP connections is degraded when multiple TCP connections with different propagation delays share the single bottleneck link. For instance, we have shown that the fairness among TCP connections becomes worse as the packet processing delay in the router's buffer becomes smaller.

Our analytic approach has advantage over other analytic approaches[7, 8] since our analytic approach can be extended to analyze the transient state behavior of TCP. We are currently working on the transient state analysis of TCP when multiple TCP connections have different propagation delays.

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