Analyzing and modeling router-level internet topology and application to routing control
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ABSTRACT

Measurement studies on the Internet topology show that connectivities of nodes exhibit power-law attribute, but it is apparent that only the degree distribution does not determine the network structure, and especially true when we study the network-related control like routing control. In this paper, we first reveal structures of the router-level topologies using the working ISP networks, which clearly indicates ISP topologies are highly clustered; a node connects two or more nodes that also connected each other, while not in the existing modeling approaches. Based on this observation, we develop a new realistic modeling method for generating router-level topologies. In our method, when a new node joins the network, the node likely connects to the nearest nodes. In addition, we add the new links based on the node utilization in the topology, which corresponds to an enhancement of network equipments in ISP networks. With appropriate parameters, important metrics, such as the a clustering coefficient and the amount of traffic that pass through nodes, exhibit the similar value of the actual ISP topology while keeping the degree distribution of resulting topology to follow power-law. We then apply the routing control method to the ISP topologies and show that the optimal routing method gives much smaller maximum link utilization (about 1/3) compared with the minimum hop routing which is often used in the operating networks. Accordingly, we examine a heuristic routing method suitable to the ISP topologies with consideration of technology constraints of IP routers. The evaluation results show that our modeling method can be actually used for evaluations on routing control.

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1. Introduction

Recent measurement studies on Internet topology show that the connectivities of nodes exhibit a power-law attribute (e.g., see [1,2]). That is, the probability p(k) that a node is connected to k other nodes follows p(k) \sim k^{-\gamma}. In recent years, considerable numbers of studies have investigated power-law networks whose degree distributions follow the power-law [3–8]. Here, the degree is defined as the number of out-going links at a node. The theoretical foundation for the power-law network is introduced in Ref. [9] where they also presents the Barabashi–Albert (BA) model in which the topology increases incrementally and links are placed based on the connectivities of topologies in order to form power-law networks. The resulting power-law networks have two main characteristics: (1) a small number of links are connected with numerous nodes, while a large number of links are connected with a few nodes, and (2) the number of hop-counts between nodes is small (small-world property).

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However, even if the degree distributions of some topologies are the same, more detailed characteristics are often quite different. A pioneering work by Li et al. [10] has enumerated various topologies with the same degree distributions, and has shown the relation between the characteristics and performances of these topologies. With the technology constraints imposed by routers, the degree of nodes limits the capacity of links that are connected to. Li et al. point out that higher-degree nodes tend to be located at the edges of a network, and they then demonstrate in an Abilene-based topology where the power-law network can actually be constructed by maximizing the throughput of the network with the technology constraints imposed by routers. Their modeling method in [10] provides a new insight in that the location of higher-degree nodes are not always located at the core of networks. Actually, different to AS-level topology, each ISP constructs its own router-level topology based on strategies such as minimizing of the mileage of links, redundancies, and traffic demands. In this sense, their method is based on a strategy that maximizes the utilization of IP routers under current router technology constraints.

However, the Abilene network used in Ref. [10], which is one of scientific networks, is different to other ISP networks as will be discussed in more detail in Section 3. The main difference may come
from the fact that scientific networks like Abilene provide fewer opportunities to enhance their network equipment because of budgetary constraints, while ISPs make their efforts on enhancement of networks based on their strategies. This can clearly be seen from the graphs of the Abilene network (Fig. 6(e) of Ref. [10]) and the Sprint network (Figs. 7 and 8 of Ref. [11]). One of our main motivations in this paper is to investigate the modeling methodology for actual ISP topologies. For this purpose, structural properties other than the degree distribution are important. Although Li et al.’s approach is significant, it is insufficient to explain redundant considerations in building ISP networks. As will be discussed in Section 3, the Sprint topology and Abilene-based topologies are quite different in terms of the clustering coefficient. More importantly, these differences greatly affect methods of network control. One typical example is routing control as we will demonstrate it in Section 3.3; the link utilization in the router-level topology is much far from the one in the conventional modeling method. The same argument could also be applied to the higher-layer protocols. That is, for vital network researches, a modeling method for a realistic router-level topology is urgently needed to be developed [12,13], which is our next concern.

In this paper, we develop a modeling method to construct ISP router-level topologies. To achieve this, we first reveal basic structures for the router-level topologies other than the power-law property of degree distribution. The results clearly reveal the ISP topologies had a much higher clustering coefficient than the AS topology [14], the topology examined by Li et al. [10], and the other topologies attained with conventional modeling methods. We examine how these structural differences affects throughput performance in the next step using minimum hop routing and optimal routing methods. The results show that at the topologies by the BA model, the optimal routing method gives much smaller the maximum link utilization than the minimum hop routing, in Section 5, we present and examine a routing method that achieve lower link utilization in router-level topologies. Section 6 concludes the paper.

2. Related work

In the Internet, the degree distribution has been shown to follow the power-law at the AS-level [1,2,15]. Barabasi and Albert [9] presents a BA model in which the topology grows incrementally and links are attached to nodes based on a preferential probability, \( P_i = d_i / \sum_{j} d_j \), where \( d_i \) is the degree of node \( i \). The characteristic of topologies attained with the BA model is further investigated by other researchers [6].

Bu and Towsley [14] compares the structure of the BA model with AS-level topology. Their results show that degree distribution as well as the clustering coefficient with the BA model does not match those with the AS topology because new ASs have a stronger preference for hub nodes compared to the linear preference used with the BA model. They then propose a non-linear preferential probability, \( P_i = (d_i - \beta) / \sum_{j} (d_j - \beta) \), to generate AS-like topologies. \( \beta < 1 \) is a parameter that increase the preferential probability for high-degree nodes. Zhou and Mondragon [16] incorporate an emergence of new internal links between existing nodes in addition to the non-linear preferential probability. Haddadi et al. [17] present parameter settings for topology generator tools, such as BRITE, TIERS, and GT-ITM, to generate realistic AS-level topologies.

Fabrikant et al. [18] presents an FKP model for generating a power law graph. This uses the incremental growth model, but the cost for link attachment is different to that for the BA model. The researchers introduce two distance-related metrics for the attachment: the physical distance of nodes, \( d_{i,j} \), and the hop-distance to an initial or “root” node. The cost of attachment is the sum of these two metrics, but the physical distance is weighted by \( \alpha \). Depending on the value of \( \alpha \), the resulting topology creates a phase transition between the star, exponential, and power-law graphs. The FKP model is further generalized in [19] so that AS-like topologies can be generated.

In addition to topological modeling for AS-level topologies, several researches focus on flow-level behavior. Goh et al. [6] pointed out that, under minimum hop routing, the distribution in the number of node-pairs that pass through node \( i \) follows the power-law, \( P_i(l) \sim l^{-\gamma} \). Gkantsidis et al. [5] derive the lower bound of “congestion”, which is defined as the maximum number of demands that pass through a link in a power-law network. They show that when an approximate multi commodity max-flow min-cut theorem is used, the congestion scales as \( O(n \log n) \) where \( n \) is the number of nodes. Akella et al. [3] shows how the congestion scales as \( n \) increases when single shortest path routing is used. The simulation and analytical results revealed that the congestion scales as \( \Omega(n^{1-1/k-1}) \), which implies that the congestion increases linearly as the number of nodes \( n \) increases. They also demonstrated that BGP’s policy routing [20] marginally better than the minimum hop routing. However, all these works focus on the AS-level topology.

There are relatively few studies on modeling router-level Internet topology. Heckman et al. [21] present parameter settings for topology generator tools, such as BRITE, TIERS, and GT-ITM, to construct ISP topologies. However, the topology they examined is a POP (point-of-presence) level topology. Quotin et al. presents IGP topology generator for generating more realistic router-level topologies [22]. However, it requires more detailed information on router-level topologies, such as geographical information and number of routers in a PoP. Li et al. [10] enumerated various topologies with the same degree distributions, and showed the relation...
between their structure and performances of those topologies. They pointed out that because of a technology constraint of commercial routers, high-degree nodes accommodate low-bandwidth access lines while lower-degree nodes accommodate high-bandwidth core lines because of technology constraints with commercial routers. Due to technology constraints, the hub node is located at the edges of the network, while in the AS-level topology the hub node is located at the core of the network. With a three-level hierarchical structure based on the Abilene network and the previously mentioned link capacity constraints, Li et al. show that there exists a topology such that the throughput of the topology is maximized while the degree distribution follows a power-law. In the Abilene-based topology presented in Fig. 6(e) of Ref. [10], there are no redundant links between nodes (except in network cores), and a single node/link failure will easily split the network, while the ISP topologies presented in Ref. [11] clearly include redundant links. Therefore, we cannot apply their modeling method to traffic flow level researches like routing control.

3. Basic properties of router-level topology

In this section, we investigate the structure of router-level topologies as a first step to modeling a router-level topology, and discuss the differences between actual ISP’s router-level topologies and topologies generated by existing modeling methods.

3.1. Network Motif

Milo et al. [23] have introduced the concept of Network Motif. The basic idea is to find several simple structures in complex networks. In this paper, we select four-node subgraphs as building blocks for router-level topologies following the Milo et al.’s approach, i.e., rectangular (Fig. 1(a)), tandem (Fig. 1(b)), sector (Fig. 1(c)), umbrella (Fig. 1(d)), full-mesh. The case of a three-node subgraph, which has an exactly the same meaning as “cluster”, will be discussed later. Fig. 2 plots the frequency of four-node subgraphs appearing in each topology. The labels along the horizontal axis represent the ISP networks (from ISP1 to ISP7) that have been measured with Rocketfuel tools [24, 11]. A topology generated by the BA model (Model1), such that the number of nodes and links is the same as that for the Sprint topology is also presented. For obtaining the BA topology, the preferential attachment and incremental growth are applied. Note that the average degree of BA (and Sprint) topology is 3.087, which cannot generate by the original BA model where the pre-specified number of links are attached upon node arrival. In order to obtain the same number of nodes and links to the Sprint topology, three new links are attached upon node arrival, and with a little probability, one more link is attached. The probability is adjusted such that the expected number of links (45 links for the current case) is the same to the Sprint topology.

The results from the Abilene-based topology used in Ref. [10] (Model2) is also plotted in the figure. Table 1 summarizes the numbers of nodes and links of these topologies. We can see that: (1) there are many more “sectors” with the Sprint topology (ISP1) than with the BA topology (Model1), (2) “full-mesh” appears more often than model topologies in the router-level topologies of ISPs (Sprint, abovenet, AT&T, ebone exodus, level3, verrio), (3) the percentile sum for “rectangle”, “umbrella”, and “sector” is large (around 30%) for ISP topologies while not for model topologies.

From the figure, it is quite apparent that router-level topology is very different to the topologies generated with conventional modeling methods. Furthermore, ISP-level topologies (from ISP1 to ISP7) are highly clustered compared with the Abilene-based topology (Model2) presented by Li et al. [10]. We conjecture that the reason for differences derives from redundancy considerations in building the ISP networks. In what follows, we concentrate on the Sprint Topology (ISP1) and investigate the router-level topology in detail.

3.2. Detailed analysis of router-level topology

To compare how the previously-discussed structure for router-level topology affects the basic properties of networks, we prepare three topologies that have the same number of nodes and links. For the router-level topology, we use ISP1 (Sprint). Two topologies generated by the BA model (Model1 in Fig. 2) and the ER topology generated by the ER model [25] in which links are randomly and also used for purposes of comparison. The degree distributions for these topologies are shown in Figs. 3(a)–(c). From Fig. 3(a), we can confirm that the degree distribution for the Sprint topology follows a power-law.

We use the following metrics for node $i$ to investigate the characteristics of topologies:

- $A(i)$, $D(i)$: Average and maximum number of hop-counts from node $i$ to all other nodes. Hereafter, we will call the maximum hop-counts as diameters.
- $C_v(i)$: Clustering coefficient [26] for a node, which is defined as

\[
C_v(i) = \frac{2E_i}{d_i(d_i - 1)},
\]

where $d_i$ is the degree of node $i$, and $E_i$ is the number of links connected between node $i$’s neighbor nodes.

We also consider two centrality measures: degree centrality and node betweenness centrality [27]. For each node $i$, degree centrality is defined as the degree of node $i$, and node betweenness centrality is defined as the number of node-pairs that pass through node $i$. We calculate the node betweenness centrality by assuming the minimum hop routing and normalize the number of node-pairs that pass through node $i$ by the number of possible node-pairs, i.e., $N \times N - 1$ for $N$-node topology, so that the node betweenness centrality ranges from 0 to 1. Note that the node betweenness centrality does not reflect the actual traffic demand on the node.
Nevertheless, we use the node betweenness centrality to characterize ISP topologies because it gives a fundamental characteristic to identify the amount of traffic flow on topologies. For simplicity, hereafter, we refer to the node betweenness centrality as betweenness centrality.

![Fig. 2. Distribution of four-node subgraphs.](image)

Table 1  
Number of nodes and links of topologies.

<table>
<thead>
<tr>
<th>ISP1</th>
<th>ISP2</th>
<th>ISP3</th>
<th>ISP4</th>
<th>ISP5</th>
<th>ISP6</th>
<th>ISP7</th>
<th>Model1</th>
<th>Model2</th>
</tr>
</thead>
<tbody>
<tr>
<td># of nodes</td>
<td>516</td>
<td>366</td>
<td>729</td>
<td>159</td>
<td>244</td>
<td>624</td>
<td>893</td>
<td>516</td>
</tr>
<tr>
<td># of links</td>
<td>1593</td>
<td>966</td>
<td>2253</td>
<td>307</td>
<td>540</td>
<td>5298</td>
<td>2217</td>
<td>1593</td>
</tr>
</tbody>
</table>

![Fig. 3. Degree distribution.](image)

(a) Sprint topology  
(b) BA topology  
(c) ER topology

The clustering coefficient for each node is ranked in ascending order in Fig. 4(a). In the figure, the results of the Abilene topology are also presented. We can see that the clustering coefficient for the Sprint topology is much larger than that for the BA topology, as we expected from Fig. 2. We also show the relation between
the clustering coefficient and the other centrality measures in Fig. 4(b) to (e). Note that the reason for sorting by clustering coefficient is that the clustering coefficient captures the results of Fig. 2 and is easy to distinguish whether the modeling method presented in Section 4 will capture the structure of ISP topology or not.

Furthermore, the results in Figs. 4(a) and (d) show that lower-degree nodes are more highly clustered with the Sprint topology; a node with two out-going links always forms a cluster, while higher-degree nodes do not always have a high clustering coefficient. Other interesting observations can be seen in Figs. 4(b) and (c); the diameter \( D(i) \) and the average distance \( A(i) \) of the Sprint topology are relatively larger than those of the BA topology. A node in the BA model tends to be connected to higher-degree nodes, and therefore any two nodes communicate with smaller hop-counts via the higher-degree nodes. However, the average distance with the Sprint topology is larger than that with the BA topology. Therefore, another attachment metric, rather than the degree-based metric, has to be considered to model the router-level topology, which we will discuss and propose in Section 4. The Abilene topology shows quite different characteristics in Fig. 4(a). With the Abilene topology, the clustering coefficient is even lower than the BA topology, and the average path length is much longer than the Sprint topology and the BA topology. The reason for this is apparent in that the Abilene topology is three-level hierarchical topology.

As previously discussed, the structure of router-level, especially ISP-level topologies, is very different from the BA and Abilene topologies. In the next section, we evaluate how these structural differences affect the performance of networks.

3.3. Evaluation of load properties on the router level topology

The load characteristics in router-level topology are important for evaluating route control mechanism. In this section, we discuss our evaluation of routing methods for three topologies that have the same number of nodes and links. To achieve this, we use minimum hop routing and optimal routing. Optimal routing is based on the flow deviation method [28].

3.3.1. Simulation models

To clarify the load characteristics in the router-level topology, we again use the Sprint topology (ISP1), the BA topology (Model1), and the ER topology described in Section 3.2. The Abilene topology
is not compared here since the topology has few redundant links and thus cannot be applied to evaluating routing methods.

In the simulation, each node-pair generates the same amount of traffic at a unit time in these three topologies. We assume that the processing performance for each node would be 320 Gbps, which could be provided by Cisco 12416 [29]. As Li et al. mentioned [10], constraints with router technology limit the degree (i.e., number of ports in the router) and line speed of a port. We therefore allocate the capacities of links based on the Cisco 12416 specification. The details are described in Section 3.3.3.

3.3.2. Optimal routing method

To evaluate the load characteristic on the router-level topology, we use a flow deviation method [28] to obtain the optimal link load. After that, we compare minimum hop routing and optimal results. The flow deviation method with the minimum hop routing, which is the most popular routing control method in the Internet. The flow deviation method incrementally changes the flow assignment along feasible and descent directions. Given objective function \( T \), the method set \( x_l \) as a partial derivative of the function \( T \) with respect to \( F_l \), where \( F_l \) is the amount of traffic that traverses link \( l \). Then, the new flow assignment is solved by using the shortest path algorithm in terms of \( x_l \). By incrementally changing from the old to the new flow assignment, optimal flow assignment is determined. In this study, we set objective function \( T \) to

\[
T = \sum_l 1/(C_l - F_l),
\]

where \( C_l \) is the capacity of link \( l \) and \( F_l \) is as defined above.

3.3.3. Method for allocating link capacities

We allocate the capacities of links based on the technology constraints imposed by the Cisco 12416 router, which has 16 line card slots. When a router has 16 or less connected links, all the links can have 10 Gbps capacity. If there are more than 16 links connected to the router, the capacity for one or more of the links should be decreased [10].

However, it is difficult to determine in which link capacity should be decreased. Therefore, we allocate the capacities of links in a network so that the amount of traffic between a node-pair is maximized, while satisfying the following two technology constraints imposed by routers.

- The capacity of a link is chosen from a set \([100 \text{ Mbps}, 1 \text{ Gbps}, 2.4 \text{ Gbps}, 4.8 \text{ Gbps}, 10 \text{ Gbps}]\);
- Each router can handle the traffic up to 320 Gbps. That is, the total capacity of links connected with the router is 320 Gbps or less.

The first constraint corresponds to the link capacity constraint on routers; the set is chosen from the Ethernet technology for 100 Mbps and 1 Gbps, and optical transmission technology from 2.4 Gbps to 10 Gbps. The second constraint represents node capacity constraint on routers. Under these constraints, a router accommodates several low speed tributaries, i.e., it has more than 16 out-going links in the current case, unless the total capacity of the links violates the second constraint.

The algorithm for allocating link capacities is as follows. We give an amount of traffic between nodes \( i \) and \( j, d_{ij} \), as input values. Each node-pair generates the same amount of traffic at a unit time in the above-mentioned three topologies, that is, we set the identical value \( d \) to \( d_{ij} \). Then, we check whether the node capacity constraint is satisfied. If the constraint is violated, we decrease the capacity of links such that the link capacity constraint is satisfied. If there are no allocations of link capacities that satisfy two constraints, \( d \) is decreased.

The specific procedure is as follows. Given a network topology, flow assignment, and the amount of traffic generated between nodes, \( d \), do following steps:

Step 1: Calculate the amount of flow, \( F_l \), that traverses link \( l \) by using the given flow assignment and the (identical) traffic demand between nodes \( d \).

Step 2: Initialize the link capacity; set the capacity, \( C_l \), of the link \( l \) to be 10 Gbps.

Step 3: Check whether the node capacity constraint is satisfied. For each node-pair \( ij \), repeat the following steps.

Step 3.1: Calculate the sum of the capacity of all links (denoted by \( C_{ij}^{\text{all}} \)) that are connected to node \( p \).

Step 3.2: If \( C_{ij}^{\text{all}} \) is greater than 320 Gbps, do the following steps until \( C_{ij}^{\text{all}} \leq 320 \text{ Gbps} \).

Step 3.2.1: Decrease the capacity of all links that are connected to node \( p \) by one step lower than the current capacity, e.g., 10 Gbps → 4.8 Gbps if the current link capacity is 10 Gbps. However, we do not decrease the capacity of link if decreasing it violates the link capacity constraint.

Step 3.2.2: If we cannot decrease the capacity of links any further, decrease the amount of flow between nodes \( i \) and \( j, d_{ij} \) (in the current case \( d \)), and go back to Step 0.

After this algorithm finishes, the capacity \( C_l \) for each link \( l \) is obtained. We also obtain \( d_{ij} (=d) \) as the maximum traffic demand for nodes \( i \) and \( j \) at which the network can accommodate.

3.3.4. Comparison of the maximum traffic demand between nodes

Table 2 lists the maximum traffic demand that node-pairs can transmit. The maximum link utilization in the topology with this value becomes 1.0. Let us first look at the results when minimum hop routing is applied. We can see from the table that when the amount of traffic generated from each node-pair is beyond 600 Mbps with the Sprint topology, the maximum link utilization exceeds 1.0. Thus, with the increasing deployment of Giga-bit Ethernet technology, the Sprint topology will not be able to accommodate the traffic. We can also see that 590 Mbps for each node-pair is an upper bound that the BA topology can accommodate. However, the maximum traffic demand is 6.0 Gbps with the ER topology. The reason is that since the maximum degree of nodes in the ER networks is below 16, all the corresponding links can have 10 Gbps capacity.

We next look at the results for the optimal routing method. Similarly, the ER topology can accommodate more traffic. Note that with the BA topology, the maximum traffic demand increases by a factor of 15.6, while that with the Sprint topology only does by a factor of 2.67. The reason for this is as follows. If minimum hop routing is used, the bottleneck of the BA topology is the link that is connected to the hub-node where the capacities of neighboring links are decreased by the link capacity constraint. Furthermore, lower-degree nodes, which can have the larger capacity for neighboring links, is not utilized in the BA topology. If optimal routing is used, lower-degree nodes assist to accommodate traffic. Thus, the maximum traffic demand increases greatly with the BA topology.

<table>
<thead>
<tr>
<th></th>
<th>Sprint</th>
<th>BA</th>
<th>ER</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum hop</td>
<td>600</td>
<td>590</td>
<td>6000</td>
</tr>
<tr>
<td>Optimal</td>
<td>1600</td>
<td>9190</td>
<td>14500</td>
</tr>
<tr>
<td>Ratio</td>
<td>2.67</td>
<td>15.6</td>
<td>2.42</td>
</tr>
</tbody>
</table>
3.3.5. Comparison of distribution of link utilization

Figs. 5, 6, and 7 show the distributions for link utilization. In each figure, upper one (a) shows the results of minimum hop routing, and the lower one (b) for optimal routing. The vertical axis represents link utilization, which is defined as $F_{ij}/C_i$. The horizontal axis is the degree of the source node of the corresponding link. The amount of traffic in each topology is obtained by setting $d$ such that the maximum link utilization becomes 1.0 by using the minimum hop routing method.

We can see a variety of links being utilized when minimum hop routing is used with the BA topology. Especially, the link connected with the higher-degree node is hardly used. If we use optimal routing, there are fewer variations with link utilization, and each link has almost the same utilization. As for the Sprint topology, the correlation between link utilization and the degree of source nodes is small with the Sprint topology if minimum hop routing is used. When the optimal routing is used, all link utilizations decrease as expected. However, the variations are still large, which is different from the tendency with the BA topology. The reason for this derives from the structure of the Sprint topology, where the clustering coefficient is much larger than with the BA topology. In other words, a node that is connected to another node (say node $A$) is also connect to a neighbor (or near), in terms of physical, node from node $A$. Here, congestion at some links cannot be avoided even if another link is selected, because that link is still connected to a node around the congested link. Preferential attachment in the BA model, on the other hand, does not incorporate the locality of connecting nodes, and thus optimal routing can find low-congested links.

Note that finding the low-congested link in BA topology and ER topology is not good especially when the topology is used for evaluating routing control methods; evaluation on model-based topologies over-estimates the performance of routing control methods. We therefore require a modeling method for the realistic router-level topology to evaluate routing control mechanisms. In addition, optimal routing with the Sprint topology method still reduces the maximum link utilization to 1/3, and thus a routing method that accommodates more traffic is still required in the ISP topologies.

4. Modeling methodology for router-level topologies

The results in the previous section revealed that ISP-level topologies are very different to topologies using conventional modeling methods in that: (1) the clustering coefficient for lower-degree nodes is high, and (2) improved maximum traffic demand between nodes achieved by the optimal routing with the ISP topology is less than that in the BA topology. This clearly indicates that ISP topologies are locally clustered networks, i.e., each node is connected to geographically closer nodes, and thus topologies attained by conventional models that do not use geographic information cannot appropriately evaluate for network control mechanisms, such as routing control.

Fabrikant et al.’s FKP model in Ref. [18] is a method that incorporates geographical information. However, they did not discuss in Ref. [18] whether the topologies resulting from the FKP model matches router-level topologies or not. In this section, we show that although topologies obtained with the FKP model are close to router-level topologies, they still have a lower clustering coefficient and do not match betweenness centrality. We therefore propose a new modeling method to generate router-level topologies in Section 4.2.

4.1. FKP topology: distance-based modeling

The FKP model proposed by Fabrikant et al. [18] revealed that the power-law property of degree distribution can still be obtained by minimizing “distance” metrics. This model does not use preferential attachment to add links, and instead uses minimization-based link attachment. More specifically, the FKP model works as follows. Each new node arrives at randomly in the Euclidean space $(0, 1)^2$. After arriving at new node $i$, the FKP model calculates the following equation for each node, $j$, already existing in the network:

$$\alpha \cdot w_{ij} + b_j,$$

where $w_{ij}$ is the Euclidean distance (i.e., physical distance) between nodes $i$ and $j$, and $b_j$ is the hop-counts distance between node $j$ and a pre-specified “root” node (node 0). $\alpha$ is a parameter that weights the importance of physical distance. If $\alpha$ has a lower value, each node tries to connect to higher degree nodes; $\alpha = 0$ is an extreme scenario that creates a star-topology. If $\alpha$ has a higher value, each node tries to connect their nearest nodes. A topology with high $\alpha$ is shown to behaves like an ER topology. The power-law property of the degree distribution appears at a moderate value of $\alpha$ value. Here, there are several hub-nodes in each region, and the hub-nodes form a power-law.

Fig. 8 compares the ISP topology with the FKP model with regard to the same properties we previously discussed. In the figure, we do not use the actual Sprint topology (ISP1), but we modified the Sprint topology by eliminating one-degree nodes and their corresponding link since one-degree node has no impact on routing control. The resulting topology has 439 nodes/1516 links, and the average degree is 3.46. In obtaining the results of the FKP topology, we add three links when each node arrived in order for setting the total number of links so that it is almost the same as for the modified Sprint topology. For the initial graph $G_{out}$, we use the 14-node NSFnet topology with geographic latitudinal and

![Fig. 5. Link load distribution: the ER topology.](image-url)
longitudinal information. Since we generate the topology that has more than 400 nodes, the structural differences in the initial topology will not affect the topology generated. The value for $a$ is set to 40 as used in Ref. [18].

A first impression of the results for the FKP topology is that the shape is closer than the results for the BA topology (see Fig. 4(a) through (e)). However, a clear difference appears again in the clustering coefficient; although the FKP model constructs a more highly-clustered network than the BA topology, the clustering coefficient is still smaller in lower-degree nodes. Another difference is that the maximum degree of the FKP topology is low. Note that the maximum degree depends on the parameter setting. As $a$ gets smaller, the maximum degree can be increased. However, at the same time, a smaller value of $a$ leads to a star-like topology and the betweenness centrality also becomes larger than the value in Fig. 4(e). Therefore, in the FKP model, fitting the degree distribution by appropriate $a$ results in mismatches on the betweenness centrality of the modified Sprint topology.

4.2. New modeling method for router-level topologies

The fact that the FKP model cannot construct router-level topologies because of much larger betweenness centrality drives us to develop a new modeling method by extending the FKP model. Our model incorporate the physical distance between nodes following the FKP model. However, unlike the FKP model, we also incorporate the enhancement of network equipments in ISP networks. For this, we add new links based on node utilization in the topology. However, the problem is where to place the new link. In this paper, we select a node that have the largest betweenness centrality in the network, and then attach a link between neighboring nodes. From the view point of graph theory, adding links to neighboring nodes increases to increase the clustering coefficient of the topology. From the view point of network design, on the other hand, this corresponds to improve reliability against network failures (e.g., link failures). It also corresponds to decreasing utilization of nodes in the topologies; some part of the traffic that has passed through the most utilized node is rerouted via added links.

More specifically, our algorithm works as follows. For a given initial network $G_{init}(V, E)$, when a new node joins the network, $m$ links from that node are added (network growth). Besides, $k$ links with no relation to $m$ links are added based on node utilization of the network, which corresponds to network enhancements by ISPs (network enhancement). This procedure is continued until $n$ nodes are added to the initial network. Since $m$ links and $k$ links are added to the network at each of node join, the resulting topology has $|E| + n \cdot m + k$ links, where $|E|$ is the number of links in the initial network. In the following, we explain the link attachment policy for network growth ($m$-link addition) and policy for network enhancement ($k$-link addition).

4.2.1. Network growth model

Step 0: Set the initial network.
Step 1: For each node $i \in V$ already existing in the network, calculate the attachment cost to node $i$ as

$$x \cdot w_i + \bar{h}_i,$$

where $\bar{h}_i$ is the average distance from node $i$ to the other nodes.

Step 2: Select $m$ nodes in an ascending order by Eq. (4). Then add one link to each of selected nodes.

Step 3: Go back to Step 1, until the number of nodes reaches $n$.

4.2.2. Network enhancement model

Add $k$ links via the following steps.
Step 1: Calculate betweenness centrality for each node in the network, and then select a node, $x$, that has the largest betweenness centrality in the network.

Step 2: From the set of neighbor nodes from $x$, select two nodes $y$ and $z$, that minimize, $\frac{b}{C}w_{yz} + \frac{1}{D_y}$ if $D_z > D_y$; otherwise.

where $b$ is the parameter for weighting importance to the physical distance, and $D_p$ denotes the betweenness centrality of node $p$. Note that by using the equation $1/D_p$, more traffic on node $x$ is rerouted via the link between node $y$ and $z$.

4.3. Evaluation on modeling method

We show the results with our modeling method in Fig. 9. Here, the number of joining nodes $n$ is set to 425, and we use $m = 2$, i.e., when each node arrive, two links are prepared for newly arriving node. We set $k = 649$ so that the resulting topology has the same number of nodes (439) and links (1519) as the modified Sprint topology. If a one-degree node is necessary, the original FKP model that connects one link for node arrival can be applied. For the initial graph $G_{init}$, we use the NSFnet topology with geographic latitudinal and longitudinal information. By setting parameters $x$ and $\beta$ to be 25 and 200, the resulting topology is very close to the Sprint topology for both degree distribution and betweenness centrality. Note that we show the best parameter settings for the topology that looks like the modified Sprint topology in Fig. 9. Actually, depending on $x$ and $\beta$, the topology differs from Fig. 9. To see the impact of parameter settings, we show the maximum degree dependent on $\beta$ for each $x$ in Fig. 10. Apparently, inherited parameter $x$ from the FKP model shows the same tendency as presented in Ref. [18]; as $x$ get smaller, the topology becomes a star-like topology. That is, if the maximum degree equals to $n$ (425), the topology becomes the star topology. $\beta$ also impacts on the maximum degree in the topology; the maximum degree become larger as $\beta$ gets smaller (i.e., weights on the physical distance becomes smaller). Considering that the maximum degree in the modified Sprint topology is 47, $x$ should be greater than 20 and the $\beta$ greater than 200, to generate a realistic ISP topology with a moderate maximum degree.
5. Application to routing control evaluation

In this section, we demonstrate that our modeling method can be actually used for evaluations on routing control. Firstly, we present a heuristic routing method with consideration of technology constraints of IP routers, which achieves lower link utilization in router-level topologies. We then evaluate our heuristic routing algorithm with a topology generated by our proposed modeling method.

5.1. Our routing method

In Section 3.3, we observed that the maximum link utilization is greatly decreased by the optimal method in the router-level topology. However, it is difficult for us to actually use the optimal routing method because of its huge computing time. We therefore present and examine a routing method that achieve lower link utilization in router-level topologies.

Owing to the technology constraint of routers, a capacity of link relates to the degree of corresponding nodes. When the degree of...
the node is small, a link that are connected to the node can have a large capacity. On the other hand, as the degree of the node becomes large, the capacity of corresponding links have to be reduced. Our heuristic routing method determines the routes for each node-pair by considering this technology constraint. This is similar to the approaches in Ref. [10], but different in that the authors in Ref. [10] use this fact for constructing the topology, but we use this fact for routing control.

Our routing method incrementally determines the route of each node-pair. In determining a route for a node-pair, we use the following two policies to incorporate the technology constraint of routers. The first one is to select the route to avoid the node with the higher-degree nodes. The other one is to select a link whose capacity is larger. After we obtain the route between two nodes, the remaining capacities of all links are updated based on the selected routes. The updated information is in turn used for selecting the route for other node-pairs. Details of our algorithm are as follows.

Our routing method determines the route from node \( i \) to the other nodes (denoted as \( j \) in the following steps). That is, for each node \( i \), we perform the following steps:

**Step 1:** Set the initial costs for Dijkstra’s shortest path (SP) algorithm to all links. The cost of links is set proportional to the degree of the destination node of the corresponding link.

**Step 2:** For each destination node \( j \), repeat the following sub-steps.

**Step 2.1:** Determine the route (from node \( i \)) to node \( j \) by calculating minimum cost path by Dijkstra’s SP algorithm.

**Step 2.2:** Increase the cost for links that are used by the selected route. The amount of the increase is inversely proportional to the actual link capacities.

At Step 2.2, the cost for links is increased by inversely proportional to the actual link capacities. That is, we increase the cost to some extent if the link capacity is small, so that subsequent node-pairs will not use the lower-capacity links.

### 5.2. Performance evaluation of our routing method

We evaluate our routing method by the same three topologies described in Section 3.3.1. We obtain that the maximum traffic demand between nodes, \( d \), is 1085 Mbps for the Sprint topology. Furthermore, \( d \) of the BA topology and the ER topology is 5200 Mbps and 7670 Mbps, respectively. Our heuristic routing algorithm achieves almost the half of \( d \) comparing with optimal routing (1600 Mbps). In Fig. 11(a), we show the distribution of link utilization by applying our routing method to the actual Sprint topology (the case of the topology in Section 4 is shown later). The vertical axis shows link utilization, and the horizontal axis represents link index. The link index is given in an ascending order of link utilization when the minimum hop routing method is used. Then, the link utilization of our routing method is shown for each link index. Similarly, Fig. 11(b) is the distribution of link utilization of the optimal routing. Our routing method reduces the maximum link utilization (from 1.0 by minimum hop routing) to 0.6.

For comparison purpose, we also show the results of the BA topology in Fig. 12. Fig. 11(a) and Fig. 12(a) clearly show that our routing method has the similar variety of link utilization to the one obtained by optimal routing method.

The results of the ER topology presented in Fig. 13 show much different variety of link utilization comparing to the optimal one. The reason is explained as follows. In the ER topology, since most of nodes has a small degree (less than 16), all the links have 10 Gbps link capacity. Therefore, there is no meaning that our
routing method selects the links which have larger capacities by Dijkstra’s SP algorithm. However, since the ER topology has much different degree distribution comparing with ISP topologies, our routing method works better in the actual Internet topologies.

We finally show the link utilization of the topology generated by our modeling method in Fig. 14. The results of the modified Sprint topology is also presented in Fig. 15. By comparing Fig. 14 and Fig. 15, we observe that the distribution of link utilization in our topology is quite similar to that in the modified Sprint topology for both minimum hop routing, our routing method (the upper figure (a) for each plot), and optimal routing (for figure (b)). Although our routing method exhibit much different in the ER topology, the results of Fig. 14(b) and Fig. 15(b) indicate our modeling method constructs the realistic router-level topology that can be applied for routing control.

Our proposed model for generating router-level topologies can control the clustering coefficient by changing the parameter $\beta$ in the algorithm. The generation of a triangle is explicitly stated in the Step. 2 of Section 4.2.2.

Looking at the frequency of 4-node subgraph in 2, our proposed model will generate ISP topologies that have the similar frequency to the Sprint topology such as the Exodus and Verrio. However, our proposed model fails to capture the AT&T topology that has too much “rectangle” structure because “rectangle” cannot be controlled by making triangles.

Our recent study of packet delay dynamics on ISP topologies reveals that the flow-level behavior and its performance is characterized by the modularity structure where most of nodes are connected with short-length links and a few nodes are connected with the long-length links [30]. Extending our proposed model to
generate the “rectangle” structure or modularity structure is important for topology generation, but it is left for our future work.

6. Concluding remarks

For vital network researches, a method for modeling the realistic router-level topology urgently needs to be developed. However, we have shown that the structure of ISP topologies is quite different from that of topologies achieved with conventional modeling methods. We have demonstrated that the structural difference much affect the maximum link utilization. Based on this, we have developed a new realistic modeling method for generation of router-level topologies. In our method, when a new node joins the network, it likely connects to the nearest nodes. In addition, we added new links based on node utilization in the topology, which corresponded to enhancing network equipments in ISP networks. The evaluation results have shown that our modeling method achieve a good compatibility with the Sprint topology with regards to degree distribution and the amount of traffic passing through nodes. We have also revealed that the optimal routing method gives much smaller maximum link utilization compared with the minimum hop routing in both the actual ISP topology and topologies by our modeling method. We have therefore examined a heuristic routing method taking the technology constraints of IP routers. The evaluation results have shown the similar characteristic of link utilization in the topology as well as that our routing method achieves lower maximum link utilization in the router-level topology.

In this paper, we have concentrated on the routing control as one of network control mechanisms, and have proposed a modeling method for router-level topology that can be applied to the routing control. However, for the higher-layer protocols, it may require more detailed modeling. Actually, the link capacity model used in our work is not the optimal one, which may give much impact on studies of higher-layer protocols such as flow control. One of our future works is to reveal correlation between capacity and degree, and then consider the appropriate models for link capacity assignments in router-level topologies. Furthermore, several routing controls other than minimum hop routing have already been proposed [31]. Our next work is to apply our modeling method to these route control mechanism.

References