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Modeling of P2P File Sharing with a Level-Dependent QBD Process

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Outline of the Talk

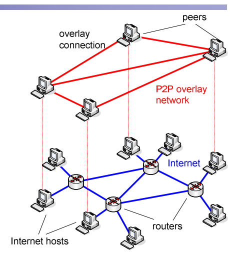
- Introduction and motivation
- P2P file sharing networks
  - The role of bandwidth sharing
  - File segmentation
- Analysis with level-dependent QBD
  - Scenario 1: all segments are lost
  - Scenario 2: any segment is lost (*catastrophe*)
- Numerical evaluation
- Conclusion and Outlook

Introduction

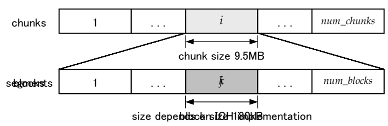
- Internet traffic volume has rapidly increased due to *peer-to-peer* (P2P)
- Common applications used for file sharing: eDonkey, BitTorrent, Kazaa, Winny
- Each peer acts simultaneously as *client and server*
- P2P networks form logical overlay topology on application layer

Peer-to-Peer File-Sharing Networks

- **Advantages:**
  - o Ease of connection setup
  - o Load is distributed among all sharing peers
    ➔ *better scalability*
  - o Flash crowds are easily compensated
- **Drawbacks:**
  - × Data may be lost due to churn
  - × Content may be manipulated by peers (*pollution/poisoning*)



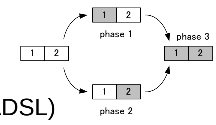
P2P System Model



- Model is similar to *eDonkey* file sharing network
- File structure consists of *chunks* and *blocks*
- Peers share blocks, corrupt chunks are discarded
- Improvement by *Intelligent Corruption Handling* (*segments*)

Download Bandwidth Sharing

- File size is F split in N_s = 2 segments
  ➔ 3 phase system
- Asymmetrical bandwidths (ADSL)
  - Upload bandwidth r_u = 128 kbps
  - Download bandwidth r_d = 768 kbps
- Fair share mechanism for upload rates lead to transition rates μ_1 and μ_2:



$$\mu_1(S,D) = \frac{1}{Z}\min\left\{\frac{S}{D}r_u, r_d\right\} \qquad \mu_2(S,D) = \frac{1}{F-Z}\min\left\{\frac{S}{D}r_u, r_d\right\}$$

Download bandwidth of peer limits transmission

S: all peers sharing segment 1
D: all peers downloading segment 1

Upload bandwidth of peer limits transmission

Analytical P2P Model

- We evaluate the ***extinction probability***, when diffusion ends due to unavailability of segments
- **Main contribution:**
  Proposal of algorithmically tractable analysis of level-dependent QBD process with application to P2P file sharing
- In the paper we consider two scenarios:
  - File diffusion stops when all segments are lost
  - File diffusion stops when any segment is lost

Markovian Process Description

The system is completely described by the following Markovian process:

$$\{(X(t), \varphi(t))\}$$

Number of peers in the system at time t — Number of peers having both segments at time t

where $\varphi(t) = (\varphi_1(t), \varphi_2(t), \varphi_3(t))$

Number of peers having segment 1 at time t — Number of peers having segment 2 at time t

with $\varphi(t)\mathbf{1} = X(t)$

Level-Dependent QBD with Catastrophes

- Extinction occurs when any segment is lost
  ➔ ***File cannot be entirely retrieved anymore***
- Two views of the transition graph:
  - Level detailed view
  - State detailed view
- Level $k \geq 1$ corresponds to set of states

$$L(k) = \{(i,j,l), i,j \in \mathbb{N}, l \in \mathbb{N}_0 \,|\, i+j+l = k\} \cup \{(i,j,0), i,j \in \mathbb{N}_0 \,|\, i+j = k\}$$

and level 0 is

$$L(0) = \{(0,0,0), (n,0,0), (0,n,0); n \in \mathbb{N}_0\}$$

Level Transitions

The system may move from level $L(k)$ to

- ***Level $L(k+1)$ with rates $A_0^{(k)}$***
  peer enters system after infinitesimal time period
- ***Level $L(k)$ with rates $A_1^{(k)}$***
  peer changes status and gets missing segment
- ***Level $L(k-1)$ with rates $A_2^{(k)}$***
  peer leaves system after infinitesimal time period
- ***Level $L(0)$ with rates $A_3^{(k)}$***
  essential peer leaves system ➔ extinction
  e.g. $(k\text{-}1, 1, 0)$, $(1, k\text{-}1, 0)$, $(0, k\text{-}1, 1)$, or $(k\text{-}1, 0, 1)$

Generator Matrix

This yields the following generator matrix Q for the Markovian process $\{(X(t), \varphi(t))\}$

$$Q = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ A_2^{(1)} & A_1^{(1)} & A_0^{(1)} & 0 & 0 & 0 & \dots \\ A_3^{(2)} & A_2^{(2)} & A_1^{(2)} & A_0^{(2)} & 0 & 0 & \dots \\ A_3^{(3)} & 0 & A_2^{(3)} & A_1^{(3)} & A_0^{(3)} & 0 & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

State Transitions

Matrix	Transitions	Rates
$A_0^{(k)}$	$(S_1, S_2, S_3) \rightarrow (S_1+1, S_2, S_3)$	$S_1 \mu_1(1, S_2+1) + S_3 \mu_2(1, S_1+S_2+1)$
	$(S_1, S_2, S_3) \rightarrow (S_1, S_2+1, S_3)$	$S_2 \mu_2(1, S_1+1) + S_3 \mu_2(1, S_1+S_2+1)$
$A_1^{(k)}$	$(S_1, S_2, S_3) \rightarrow (S_1-1, S_2, S_3+1)$	$\mu_2(S_2+S_3, S_1)$
	$(S_1, S_2, S_3) \rightarrow (S_1, S_2-1, S_3+1)$	$\mu_1(S_1+S_3, S_2)$
$A_2^{(k)}$	$S_1 > 1 \vee S_3 > 0$: $(S_1, S_2, S_3) \rightarrow (S_1-1, S_2, S_3)$	$S_1\, d$
	$S_2 > 1 \vee S_3 > 0$: $(S_1, S_2, S_3) \rightarrow (S_1, S_2-1, S_3)$	$S_2\, d$
	$(S_1 > 1 \wedge S_2 > 1) \vee S_3 > 0$: $(S_1, S_2, S_3) \rightarrow (S_1, S_2, S_3-1)$	$S_2\, d$
$A_3^{(k)}$	$S_2 > 0$: $(0, S_2, 1) \rightarrow (0, k, 0)$	d
	$S_2 > 0$: $(1, S_2, 0) \rightarrow (0, k, 0)$	d
	$S_1 > 0$: $(S_1, 0, 1) \rightarrow (k, 0, 0)$	d
	$S_1 > 0$: $(S_1, 1, 0) \rightarrow (k, 0, 0)$	d
	$(0, 0, 1) \rightarrow (0, 0, 0)$	d

Extinction Probability

- The extinction probability is computed starting from state $(0, 0, 1)$, i.e., a single sharing peer
- We define two types of matrices:
  - $(C_0^{(k)})_{ij}$ probability to reach level $L(0)$ in phase j, given that the process starts at $L(k)$ in phase i and avoids $L(1), L(2), \ldots, L(k\text{-}1)$
  - $(G_k)_{ij}$ probability to reach level $L(k\text{-}1)$ in phase j given that the process starts from level $L(k)$ in phase I
- The quantity of interest is thus

$$C_0^{(1)} = G_1$$

Extinction Probability (2)

We define the following notations:

$Q_0^{(k)} = \left(-A_1^{(k)}\right)^{-1} A_0^{(k)}$ for the probability that starting from $L(k)$, the next level-transition observed is to level $L(k+1)$

$Q_2^{(k)} = \left(-A_1^{(k)}\right)^{-1} A_2^{(k)}$ for the probability that starting from $L(k)$, the next level-transition observed is to level $L(k\text{-}1)$

$Q_3^{(k)} = \left(-A_1^{(k)}\right)^{-1} A_3^{(k)}$ for the probability that starting from $L(k)$, the next level-transition observed is to level $L(0)$

Extinction Probability (3)

- We now study matrices G_k for $k \geq 2$ satisfying:

$$G_k = \underbrace{Q_2^{(k)}}_{L(k)\to L(k-1)} + \underbrace{Q_0^{(k)}}_{L(k)\to L(k+1)} \underbrace{G_{k+1}}_{L(k+1)\rightsquigarrow L(k)} \underbrace{G_k}_{L(k)\rightsquigarrow L(k-1)}$$

one level transition

one or more level transitions

- This may be rewritten as

$$G_k = \left[I - Q_0^{(k)} G_{k+1}\right]^{-1} Q_2^{(k)}, \quad k \geq 2$$

- The ***Logarithmic Reduction*** algorithm allows computation of any G_k, $k \geq 2$, but it is sufficient to compute G_M ($M \geq 2$)
- Different for $k = 1$:

$$C_0^{(1)} = G_1 = \underbrace{Q_2^{(1)}}_{L(1)\to L(0)} + \underbrace{Q_0^{(1)}}_{L(1)\to L(2)} \left[\underbrace{G_2}_{L(2)\rightsquigarrow L(1)} \underbrace{G_1}_{L(1)\rightsquigarrow L(0)} + \underbrace{C_0^{(2)}}_{L(2)\rightsquigarrow L(0),\setminus L(1)}\right]$$

$L(0)$ is reached without returning to $L(1)$

Extinction Probability (4)

More generally, to know $C_0^{(k)}$ we first need to know $C_0^{(k+1)}$ via the general recursion

$$C_0^{(k)} = \underbrace{Q_3^{(k)}}_{L(k)\to L(0)} + \underbrace{Q_0^{(k)}}_{L(k)\to L(k+1)} \left[\underbrace{G_{k+1}}_{L(k+1)\to L(k)} \underbrace{C_0^{(k)}}_{L(k)\to L(0),\setminus\{L(1),\ldots,L(k-1)\}} + \underbrace{C_0^{(k+1)}}_{L(k+1)\to L(0),\setminus\{L(1),\ldots,L(k)\}}\right]$$

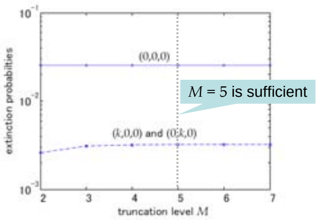
which may be rewritten as

$$C_0^{(k)} = \left[I - Q_0^{(k)} C_{k+1}\right]^{-1} \left[Q_3^{(k)} + Q_0^{(k)} C_0^{(k+1)}\right]$$

➔ Truncation of the QBD after level $L(M)$, so that

$$C_0^{(M)} = Q_3^{(M)}$$

Accuracy of Truncation Level



- File size $F = 9.28$ MB, constant death rate $d = 10^{-2}$
- Whole file extinction $(0, 0, 0)$ unaffected by M
- Truncation at $M = 5$ provides sufficient accuracy

Extinction Probability (5)

We finally obtain the following system which provides us the absorption probability $C_0^{(1)} = G_1$

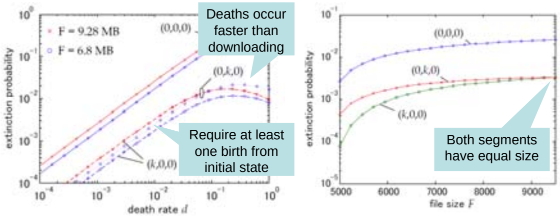
$$C_0^{(M)} = Q_3^{(M)},$$

$$C_0^{(M-1)} = \left[I - Q_0^{(M-1)} G_M\right]^{-1} \left[Q_3^{(M-1)} + Q_0^{(M-1)} C_0^{(M)}\right]$$

$$\vdots$$

$$G_1 = \left[I - Q_0^{(1)} G_2\right]^{-1} \left[Q_2^{(1)} + Q_0^{(1)} C_0^{(2)}\right]$$

Numerical Evaluation



- Smaller size of second segment leads to lower extinction probability
- Extinction probability decreases when $d \gg \mu_1(1,1)+\mu_2(1,1)$

Conclusion

- We proposed algorithmically tractable analysis of level-dependent QBD process
- Application to P2P file sharing network with segmentation
  - Peers should have long sojourn time to maintain low extinction probability
  - Content provider should offer incentives
- Future work includes derivation of further (transient) performance measures and more detailed user behavior (impatience, pollution, …)