Master's Thesis

Title

Structural analyses of router-level topologies having a power-law flow distribution

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Abstract

Modeling the Internet is vital for network researches. Recent measurement studies on the Internet topology show that the degree distribution obeys the power-law distribution. However, even if the degree distributions of some topologies are the same, detailed characteristics are often quite different. One of important factors to characterize the performance of network control methods in the Internet is flow-level characteristics, such as the amount of flow that pass through links and transmission capacity of them, because these are particular to communication networks. In this thesis, we first focus on the link capacity as a design parameter particular to realistic ISP topologies, and show that both the link capacity in the ISP's backbone network in Japan and the flow distribution obeys power-law. We then explore the reason why the flow distribution exhibits the power-law. Our results indicate that the high modularity structure of the route-level topologies is one of essential topological structures that characterize the performance of router-level topologies. We show that the origin of the modularity structure comes from minimizing the flow fluctuation on links, and show that the power-law flow distribution can result from simultaneous optimization of network throughput and tolerance against flow fluctuation.

Keyword

Power-law
ISP topology,
Link capacity
Flow distribution
Topological structure,
Modularity
Rewiring

Contents

List of Figures

List of Tables

1. Introduction

Modeling the Internet is one of important issues to evaluate network control methods [1]. Since the performance of network control methods strongly depends on Internet topology and capacity of links, a proper network model is necessary to show effectiveness of those methods. Therefore, it is important to reveal characteristics of a communication network and origin of them.

Measurement studies on the Internet topology show that the degree distribution obeys the power-law [2]. That is, the existence probability $P(k)$ of node with k outgoing links approximates to $k^{-\gamma}$ (where γ is constant). There are many modeling methods for the power-law topology [3], [4]. Among them, methods that consider technical constraints of router-level topologies are discussed in [3]. Reference [3] presents a topology generation model where a newly added node connects with existing nodes that minimize the sum of Euclidian distance between the nodes and logical distance from the existing node. The authors demonstrate that the degree distribution obeys the power-law with appropriate parameter settings. However, the topology generated by the model has too many nodes whose degree is one, and thus the topology is much different from the actual ISP topologies [5].

However, even if the degree distributions of some topologies are the same, detailed characteristics are often quite different. One of examples is the topological structure of power-law networks. A pioneering work by Li et al. [3] has enumerated various topologies with the same degree distributions, and has shown the relation between the characteristics and performances of these topologies. They point out that higher-degree nodes tend to be located at the edges of a network in router-level topology, and demonstrate that such topological characteristics maximize the throughput of the network under the technology constraints imposed by routers. Another example is the edge betweenness centrality that is defined as the number of node-pairs that passing through a link. The edge betweenness centralities of ISP topologies exhibit a power-law attribute, while the edge betweenness centrality of topologies generated by BA model does not exhibit. One of our main motivations in this thesis is to investigate the modeling methodology for realistic ISP networks. For this purpose, characteristics particular to communication networks other than the degree distribution are important.

There are relatively few studies on modeling router-level Internet topology. Heckman et al. [6] present parameter settings for topology generator tools such as BRITE, TIERS, and GT-ITM to construct POP (point-of-presence)-level ISP topologies. Fabrikant et al. presents the FKP model that incorporates geographical information to construct topologies [7]. However, they did not discuss whether the topologies resulting from the FKP model correctly models ISP topologies or not. The FKP model actually has too many one-degree nodes [8], and is very different to router-level topologies as discussed in [9]. Li et al. [3] shows the relation between their structure and performances of those topologies. The authors enumerate several topologies having the same degree distribution. Then, they evaluate the amount of traffic that the network can accommodate in each topology under the constraint of node processing capacities. The results indicate that the network throughput highly depends on the structure of topologies even though they have the same degree distribution. They pointed out that because of a technology constraint of commercial routers, high-degree nodes are deployed to accommodate low-bandwidth access lines and are located at the "edge" of networks in router-level topologies, while lower-degree nodes are used to accommodate higher line bandwidth. With a three-level hierarchical structure based on the Abilene network, they demonstrate that such the location of hub/non-hub nodes maximizes the throughput of topologies. With the technology constraints imposed by routers, correlation between degree of nodes and upper bound of node processing capacity is important to characterize the structure of router-level topologies. All of these studies investigate generation methods of router-level topologies having the power-law degree distribution. However, looking at the Figure (e) of Ref. [3], we observe that only a limited number of routers utilize their maximum processing capacities. More importantly, the structure of ISP router-level topologies, such as Sprint or AT&T topology, are still different from the structure discussed in Ref. [5], which leads to the differences in the performance of network control methods like routing [5]. Thus, it is insufficient to discuss how ISP networks are designed though Li et al.'s approach is significant.

In this thesis, we first focus on the link capacity as a design parameter particular to realistic ISP topologies, and show that link capacity design of router-level topologies and the distribution of edge betweenness centrality are essential characteristics particular to communication networks. Actually, the link capacity contributes to the cost of ISP topologies. More importantly, these differences greatly affect methods of network control. We first investigate the distribution of edge betweenness centrality in ISP topologies presented in [10]. Numerical results show that the distribution of edge betweenness centrality follows power-law and appears only on cases of router-level topologies. We then discuss the reason the distribution follows power-law in router-level topologies, and reveal that router's wire-speed architecture makes the link capacity distribution to be power-law. The wire-speed architecture of the router means that the router is composed of network-interface cards that support wire-speed packet forwarding. For example, the router is composed to have four interfaces of OC-48 (2.4Gbps) and up-link interfaces of OC-192 (10Gbps). Then, we investigate the performance and characteristics of ISP topologies when the link capacity distribution follows power-law. Our numerical results show that power-law relationship of the link capacity is essential to characterize ISP networks. We validate this observation by using disclosed information of link capacities of the IIJ (Internet Initiative Japan) network that is one of commercial ISPs in Japan. We discuss the characteristics and performance of a network designed by the wire-speed architecture and a network designed by the architecture expecting statistical multiplexing effects of links.

We next explore the reason why the flow distribution exhibits the power-law. With the power-law flow distribution, a few links that accommodate lots of flows and many with a few number of flows. As the first step, we investigate the origin of the modularity structure in ISP topologies. Ref. [3] reveals that high-degree nodes tend to connect with low degree nodes due to the technical constraints of the commercial router, and the topologies are characterized by $d2$-level information. We change the structure of the BA topology through the rewiring such that the total network throughput is maximized. The results indicate that the high modularity structure is essential and independent from the observation in Ref. [3] to characterize the topological structure of ISP topologies. We then investigate the capacity ratio that represents the additional link capacity to be tolerable against flow fluctuation. We show that there exists a tradeoff relationship between the modularity and total traffic, and the origin of the power-law flow distribution is the result by optimizing both the modularity and total traffic.

This thesis is organized as follows. In Section 2, we explain the related work on the power-law network with the power-law degree distribution. In Section 3, we investigate the structural properties of the ISP topology, and show that the modularity and the degree-degree correlation are essential to characterize the ISP topology. Section 4 discusses the flow-level distribution and its underlying topological structure. Finally, Section 5 concludes this paper.

2. Related work

2.1. Power-law Networks

It has been observed that the degree distribution of Internet topology exhibit power-law attribute [2]. Here, the power-law attribute of the degree distribution means that the probability $P(k)$ that a node is connected to k other nodes follows $k^{-\gamma}$. A theoretical foundation for the power-law network is introduced in Ref. [11] where they also present the Barabashi-Albert (BA) model to generate power–law networks. However, recent studies on Internet topology show that detailed characteristics are often quite different [3]. In this section, we describe about the typical topology models that have power-law attributes, and show some fundamental properties of power-law networks.

2.2. Network topologies and their modeling methods

2.2.1. Topologies based on modeling methods

Many studies focus on modeling methods for Internet topology. In this section, we first describe the ER (Erdos-Renyi) model in which links are randomly placed between nodes. We next introduce the BA (Barabasi-Albert) model in which the topology grows incrementally and links are placed based on the degree distribution of existing nodes, which finally forms power–law networks. We then clarify the fundamental property of power–law networks generated by BA–model

ER (Erdos–Renyi) model: The ER model was designed by Erdos and Renyi to describe communication networks. They assumed that such systems could be modeled with connected nodes of randomly placed links usually called random networks. ER model uses two parameters for generating a topology; the number of nodes N is given at first, and every two nodes are connected with the fixed probability p. Thus, the ER model generates a random network that does not have the power–law property. Using the above parameters, ER model generate a random topology by following steps

Step 1: Place N nodes.

Step 2: For each pair of nodes, put a link at probability p.

The probability $P(k)$ that a node has degree k is given as,

$$P(k) = \binom{N-1}{k} p^k (1-p)^{N-k-1}. \quad (1)$$

In addition, with large N and small p, Eq. (1) becomes,

$$P(k) = \lambda^k e^{-\lambda} / k!, \quad (2)$$

where $\lambda = pN$. From Eq. (2), the degree distribution of nodes in a random network generated by the ER model follows a Poisson distribution.

BA Model: Barabasi and Albert designed their model to emulate the growth of such large-scaled networks as the Internet. Two features characterize the BA model: Incremental Growth and Preferential Attachment. Generating a topology is started with a small number of nodes m_0.

Step 1: *Incremental Growth*: Add a new node at each step

Step 2: *Preferential attachment*: Connect the new node with m other different nodes, which are chosen with the probability Π_i such that,

$$\Pi_i(k_i) = k_i / \sum_j k_j$$

Theoretical foundations have been investigated in [11]. Reference [11] demonstrates that the BA topology has a characteristic that a small number of links are connected with numerous nodes, while a large number of links are connected with a few nodes (scale-free property). It also shows that the number of hop-counts between nodes is small (small-world property)

FKP Model: Fabrikant, et.al., have presented FKP model for generating topologies having power-law degree distribution [7]. The model also uses the incremental growth model, but rules for link attachment are different from the BA model. The FKP model does not use the preferential attachment to add links. Instead, it uses minimization-based link attachments. More specifically, FKP model works as follows.

Step 0: Place an initial node in the Euclidean space $[0, 1]^2$ at random.

Step 1: Add a new node, i, to the topology. Its location is also random in the Euclidean space $[0, 1]^2$.

Step 2: Calculates the following equation for each node, j that already exists in the network:

$$D(j) = \phi \cdot d_{ij} + h_j,$$

where d_{ij} is the Euclidean distance (i.e. physical distance) between nodes i and j, and h_j is the hop-counts distance of node j. ϕ is a constant value.

Step 3: Select a node j_0 that minimizes $D(j)$. Then connect nodes i and j_0, and go back to Step 1.

FKP model introduces two distance-related metrics for the attachment: the physical distance of nodes, d_{ij}, and the hop-distance to an initial node. The cost of attachment is the sum of these two metrics, but the physical distance is weighted by w. Depending on the value of W, the resulting topology has different characteristics. If W is a lower value, nodes seek to connect to higher-degree nodes. Especially when W=0, the resulting topology is a star-topology. If W is a higher value, new node tries to connect with its geographically close nodes. In this case, the obtained topologies behave as an ER topology that has a Poisson degree distribution. A power-law attribute of degree distribution emerges when w is a medium value. FKP model is further generalized in Ref. [9] so that AS-like topologies can be generated.

2.2.2. AS topologies

Bu and Towsley [12] compares the structure of the BA model with AS-level topology. Their results show that degree distribution as well as the cluster coefficient with the BA model does not match those with the AS topology because new ASs have a stronger preference for hub nodes compared to the linear preference used with the BA model. They then propose a new preferential probability, $\Pi'(i) = (d_i - \beta)/\sum_j (d_i - \beta)$, to generate AS-like topologies. β (<1) is a parameter that increases the preferential probability for high-degree nodes.

2.2.3. Router-level topologies

Li et al. [3] enumerated various topologies with the same degree distributions, and they showed the relationship between their structure and the performance of these topologies. They pointed out that because of a technological constraint in commercial routers, high-degree nodes accommodate low-bandwidth access lines, while lower-degree nodes accommodate high-bandwidth core lines because of technological constraints with commercial routers. When we consider such link capacity constraints, topologies based on the BA model show poor throughput due to technological constraints. That is, because hub nodes tend to be connected each other, low-bandwidth access lines between hub nodes will be a bottleneck in the network. With a three-level hierarchical structure based on the Abilene network and the previously mentioned link capacity constraints, Li et al. show a case where throughput of a topology is maximized while the degree distribution follows a power law. Although Li et al.'s approach is significant, the router-level topologies in the Internet and Abilene-based topologies are quite different in terms of the cluster coefficient. More importantly, these differences greatly affect the methods of network control. One typical example is routing control; the link utilization in the router-level topologies is much far from the one in the conventional modeling method [5].

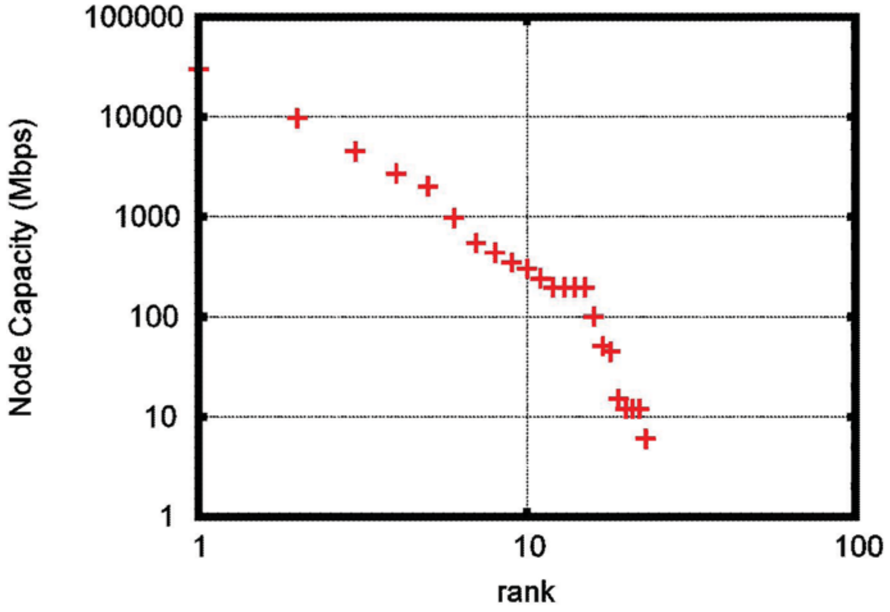


Figure 1. Node processing capacity distribution in the IIJ network

2.3. Link capacity distribution in ISP's router-level topology in Japan

Many researches about the modeling method of router-level topology have been discussed based on the characteristic that the degree distribution obeys the power-law. In these researches, there was an assumption that link capacities can be ignored, or they are identical. However, the performance of network strongly depends on link capacities because link capacities are characteristics particular to communication networks.

Ref. [13] investigates node-processing capacities of a commercial ISP network in Japan. The author uses disclosed information of link capacities in IIJ (Internet Initiative Japan) [14]. In the paper, a node processing capacity is defined as the sum of link capacity connected to the node. Figure 1, which was also presented in Ref. [13], shows the rank of node processing capacities in 2002. The vertical axis represents the node processing capacity and the horizontal axis represents the rank of that node. We observe that node processing capacity distribution obeys the power-law; its exponent is nearly -2.6. The result is interesting in that the power-law relationship again appears in the node processing capacity. However, one question arises whether the link capacity distribution follows the power-law or not. To answer this, we show the link capacity distribution in Figure 2; the result shows that the exponent of the link capacity distribution (for 40 higher-ranking) is -1. That is, the distribution obeys the zip's law. The figure is obtained by using the information on IIJ backbone network in 2002, thus it is uncertain that other ISP topologies have the observation that the link capacity distribution obeys the zip's law. However, we believe that the observation is general because we can easily derive it by the non-blocking configuration of routers. Supposing that there is one uplink port with the capacity of X in a router and there are α numbers of links, then if the router to be

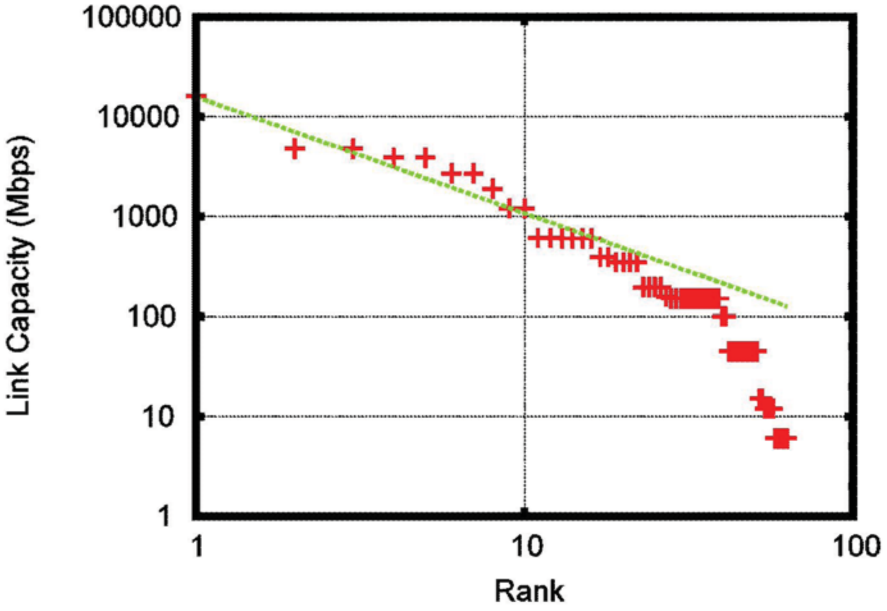


Figure 2. Link capacity distribution in the IIJ network

non-blocking, the capacity of each link should be $X = X/\alpha$. With this case, the exponent of link capacity distribution becomes -1. Base of this discussion, in the next section, we investigate the performance of networks when the link capacity distribution obeys the power-law with exponent -1.

3. Structural properties in ISP's router-level topologies

3.1. ISP's router-level topologies

We investigate flow-level properties in several router-level topologies. As ISP's router-level topologies, we use AT&T topology (729 nodes, 2253 links) and Sprint topology (516 nodes, 1593 links) provided by [10]. The Rocketfuel is a topology measurement tool based on traceroute. Note that the authors pointed out that the Rocketfuel might not cover some parts of ISP topologies. Even when the Rocketfuel misses some of routers, it is sufficient to investigate traffic dynamics because the Rocketfuel provides active paths between routers. The actual ISP networks may have some routers invisible from traceroute. We believe that the routers are used for a backup purpose because they are placed on the inactive path for Internet users. Another router-level topology is the abilene-inspired topology from Ref. [3] having 869 nodes and 1754 links. In addition, the BA topology (516 nodes, 1593 links) generated by BA model [11] is prepared for comparison purpose.

3.2. Node functionality

As discussed in Ref. [3], the network's design principles greatly affect the structure of ISP topologies. Design principles determine a node functionality [15], which in turn determines the linking of nodes. In this thesis, we consider the location of nodes in the topology. Here, the location does not mean the physical location, but the logical (in terms of hop counts) location. For each node (denoted as i), we define the following two metrics to identify the node functionality;

$$Z_i = \frac{k_i - \bar{k}}{\sigma_k}$$

where k_i is the degree of nodes, $\bar{k}$ represents the average degree in the topology, and σ_k is the variance of the degree distribution. We also define the location-related metric H as follows.

$$H_i = \frac{d_i - \bar{d}}{-\sigma_d}$$

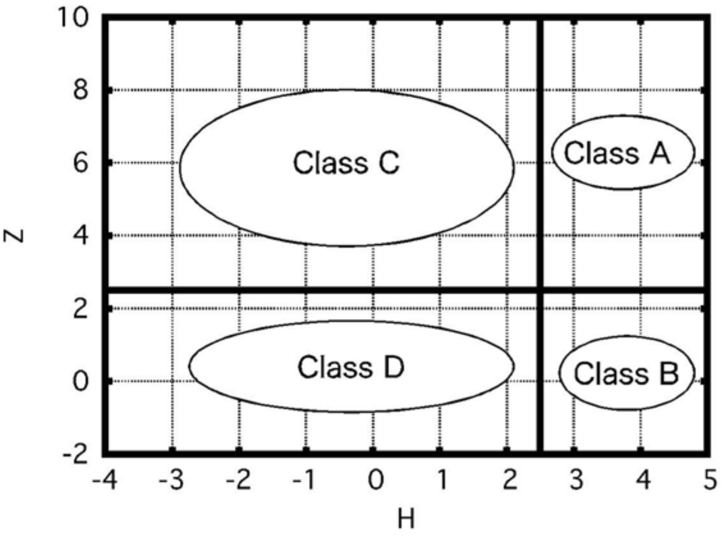


Figure 3. Classification of node functionalities

Table 1. Node Functionality

	Z_i	H_i	Functionality
Class A	high	high	Hub-core
Class B	low	high	Non-hub core
Class C	high	low	Provincial hub
Class D	low	low	Leaf (non-hub)

where d_i is the average of the hop-count distance starting from node i to the other nodes, $\bar{d}$ is the average of d_i, and σ_d is the variance of d_i. We classify the node functionality according to the value of Z and H. We then define the node functionality in Table 1. base on the fact that we can consider that the nodes whose Z_i is greater than 2.58 to be high-degree (hub) nodes. That is, when we assume the standard normal distribution for Z_i, top 0.5% nodes are hub nodes and the remaining nodes are non-hub nodes. Note that the distribution of Z_i is not always the normal distribution. However, we use 2.58 to distinguish hub nodes having a much larger number of out-going links in the module and non-hub nodes. The nodes whose H_i is greater than 2.58 are considered located at the “center” of the networks.

We then define the node functionality in Figure 3 and Table 1 base on the fact that we can consider that the nodes whose Z is greater than 2.5 to be high-degree (hub) nodes, while the nodes whose H is greater than 2.5 are considered to be located at the “center” of the networks. For the illustrative example of each class, please refer the Figure 5.

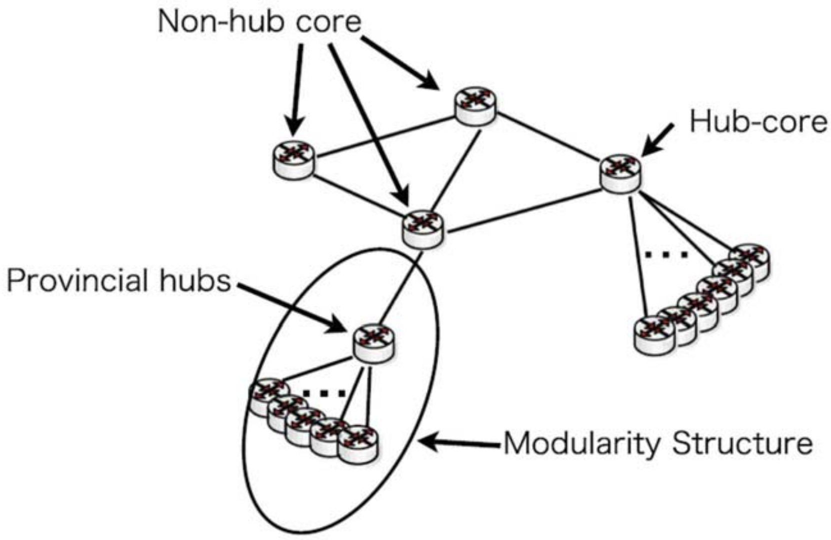


Figure 5. Illustrative example of node functionalities

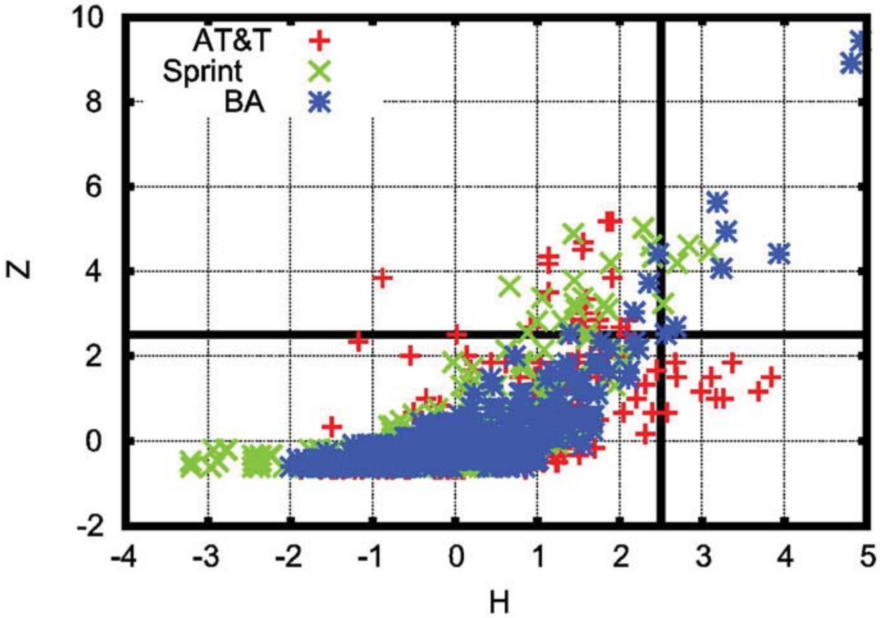


Figure 4. Node functionality in ISP topologies

Illustrative example of node functionalities

Figure 4 plots the node functionality in each topology. The BA topology has few number of "hub-core" nodes, i.e., some nodes have lots of out-going links and are located at the center of the topology. As Ref. [11] discusses, a random failure of node will mostly remove low-degree nodes, with little effect on the network connectivity. ISP topologies have several "provincial hub" nodes. A "provincial hub" node and its neighbor nodes make a modularity structure. We will discuss the modularity structure of ISP topologies in in the next section.

3.3. Modularity

Newman et al. [16] defined a modularity value (Q, $0 < Q < 1$) as,

$$Q = \sum_i (e_{ii} - a_i^2),$$

where e_{ii} is the fraction of links in the network that connect vertices in the same module, and a_i represents the fraction of links that connect between different modules. According to this definition, the modularity value of the BA topology is nearly 0.63, and that of the AT&T topology is about 0.89. This result indicates that a high-modularity structure reduces the number of highly fluctuating links.

3.4. Degree correlations

Ref. [17] introduces a dK-randomization method that rewires links randomly while keeping the distribution of degree correlation between K neighboring nodes. K is a parameter that specifies the degree of correlation from the original topology. Ref. [17] also introduces a dK-targeting dK-preserving rewriting method for generating topologies that have a structure resembling the original topology. K is a parameter that specifies the degree of correlation from the original topology; the correlation of degree between K nodes is identical to the original topology. When K is 0, the average degree is identical to the original topology. Taking K=1 leads to the same degree distribution. In the case of K=2, the probability that two nodes having degree k' and degree k'' is identical to the original topology. As K increases, a topology that more closely resembles the original topology is generated. The results indicate the average hop counts between nodes and other topological properties are mostly identical to the router-level topologies.

4. Structural properties and Flow distribution in ISP's router-level topologies

4.1. Flow distribution in ISP's router-level topologies

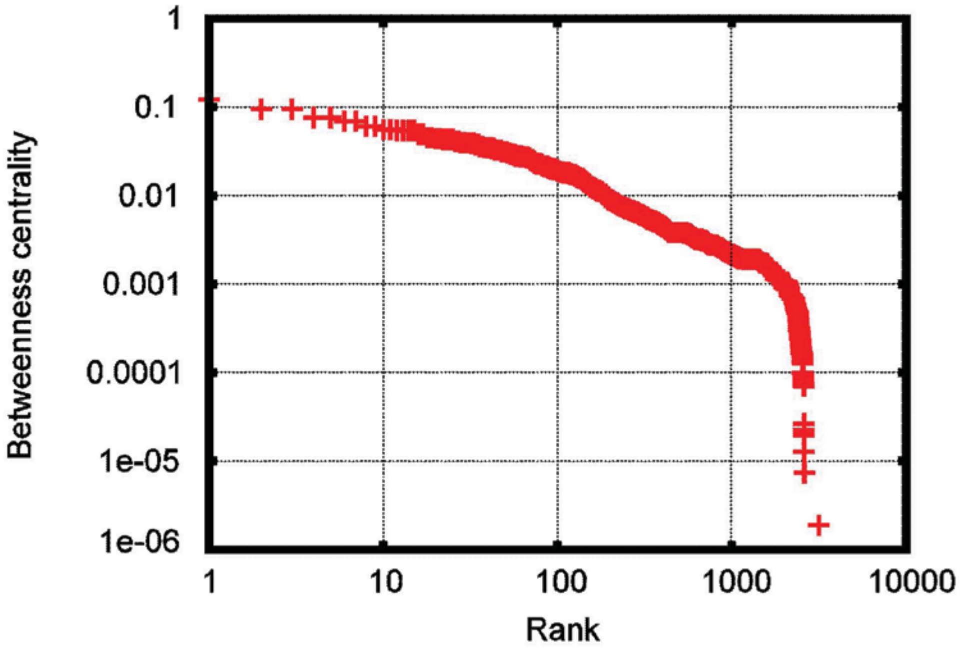
In this section, we evaluate the edge betweenness centrality in router-level topologies and a model-based topology. The edge betweenness centrality indicates for each link to determine the number of shortest paths passing through the links. Formally, the edge betweenness centrality of link e, $B(e)$, is,

$$B(e) = \sum_{s,t:s\neq t} \frac{\sigma_{st}(e)}{\sigma_{st}},$$

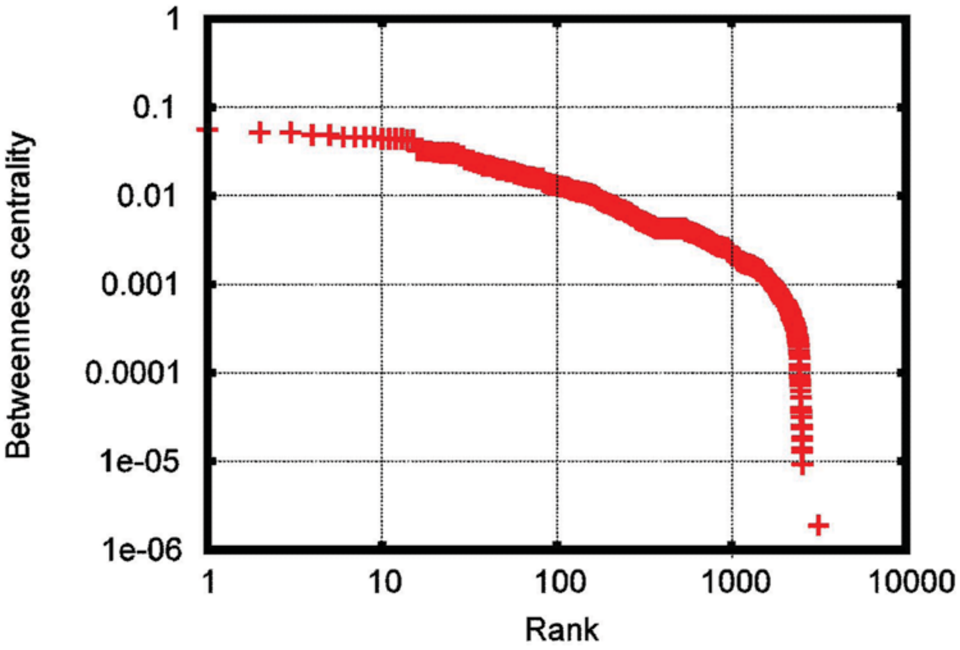
where σ_{st} is the number of shortest paths between nodes s and t, and $\sigma_{st}(e)$ is the number of shortest paths passing through the link e. As this equation indicates, the edge betweenness centrality does not reflect the amount of flow between nodes s and t. In this thesis, however, we assume that the amount of flow between nodes is identical for simplicity, and use the edge betweenness centrality as the amount of flow passing through links. The information on amount of flow passing through the links may be necessary if we concern the methods of network control, such as TCP.

Figure 6 and Figure 7 show the rank of edge betweenness centrality in each topology. Figure 6 is the results of the AT&T topology and Sprint topology, and Figure 7 for model-based topologies. From results of the ISP's router-level topologies, we observe that the rank of edge betweenness centrality shows power-law like behaviors among higher-rank and moderate-rank links. The rank of edge betweenness centrality in BA topology exhibits exponential distribution rather than the power-law distribution. A clear difference appears in Figure 6 and Figure 7 (a); the edge betweenness centrality of router-level topology follows power-law with exponent -1, while that of BA topology does not follow power-law.

These results show that the router-level topology and the realistic ISP topologies commonly exhibit power-law attribute. Then, a question is whether these results are appeared by design of ISP networks or not. For this purpose, we consider the wire-speed architecture of routers. Here, the wire-speed architecture of the router means that the router is composed of network-interface cards that support wire-speed packet forwarding inside the router. For example, the router is composed to have four interfaces of OC-48 (2.4Gbps) and up-link interfaces of OC-192 (10Gbps). Then, the traffic coming from OC-48 interfaces is multiplexed into the OC-192 interface. Assuming that there is no bottleneck inside the switching fabric inside the router, the router achieves wire-speed packet forwarding. In this case, the distribution of link capacity apparently exhibits a power-law attribute with exponent -1, which agrees with the result of router-level topologies in Figure 2. According to this observation, we next investigate the performance and characteristics of ISP topologies when the link capacity distribution follows power-law.

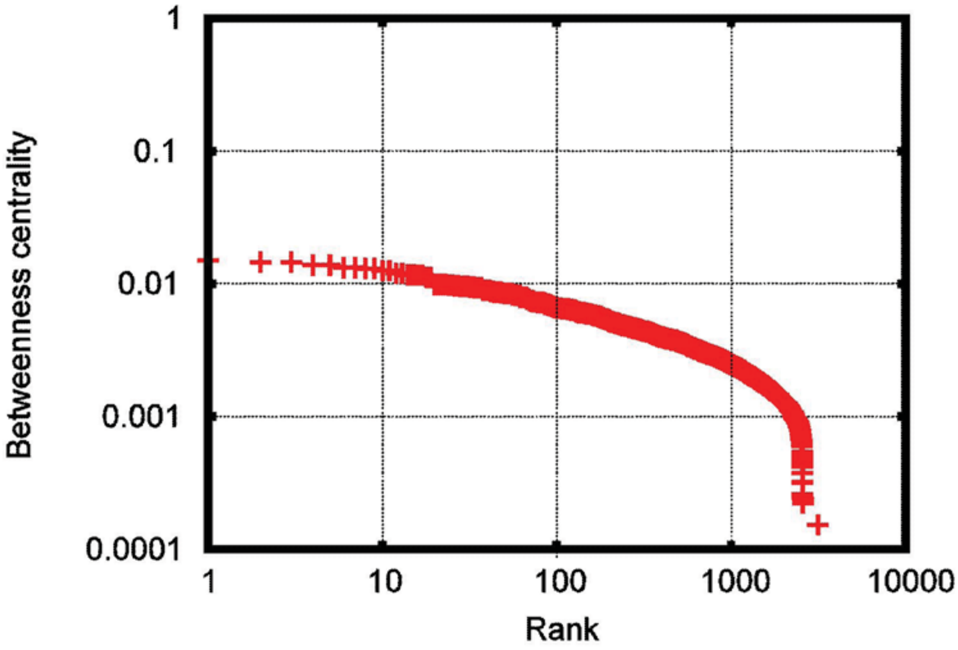


(a) AT&T topology

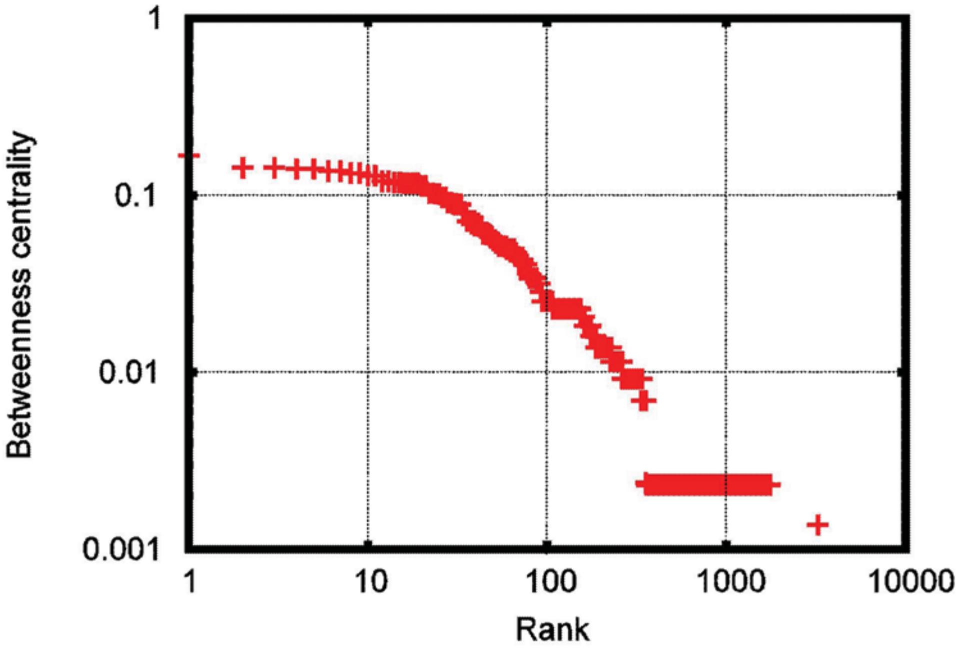


(b) Sprint topology

Figure 6. The rank of edge betweenness centrality of ISP topologies



(a) BA topology



(b) Abilene topology

Figure 7. The rank of edge betweenness centrality of model-based topologies

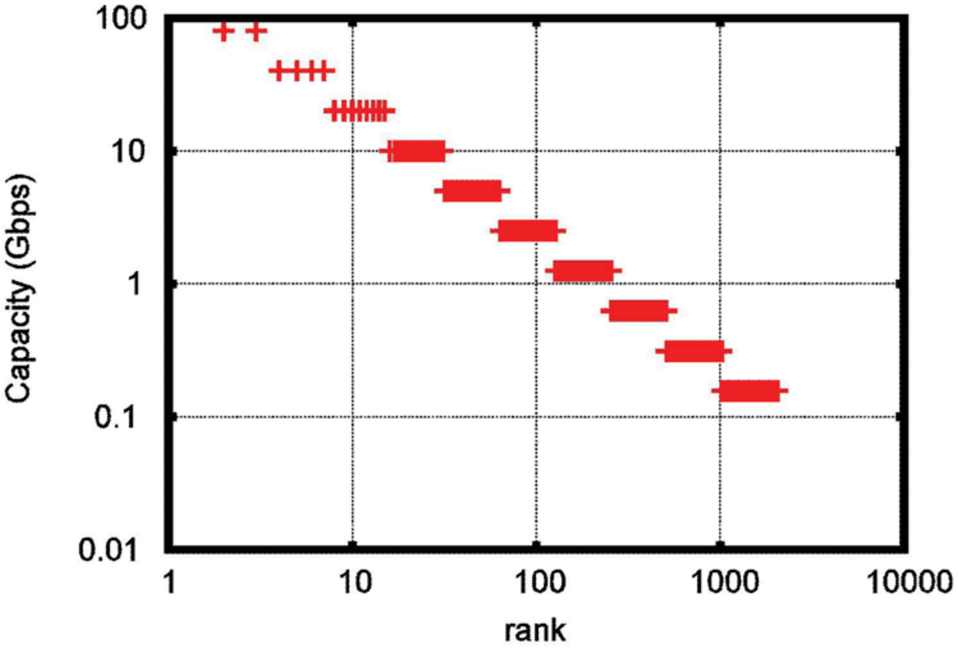


Figure 8. Link capacity distribution: B_0 = 40 Gbps, C_0=4, α=2, β=2

4.2. Link capacity distribution in ISP networks

In the previous section, we show the edge betweenness centrality is one of characteristics of the router-level topologies, and discuss that wire-speed architecture of routers makes the link capacity distribution to be power-law. Our next concern is what if the distribution of link capacity exhibits power-law attribute with the exponent -1 to the characteristics of router-level topologies.

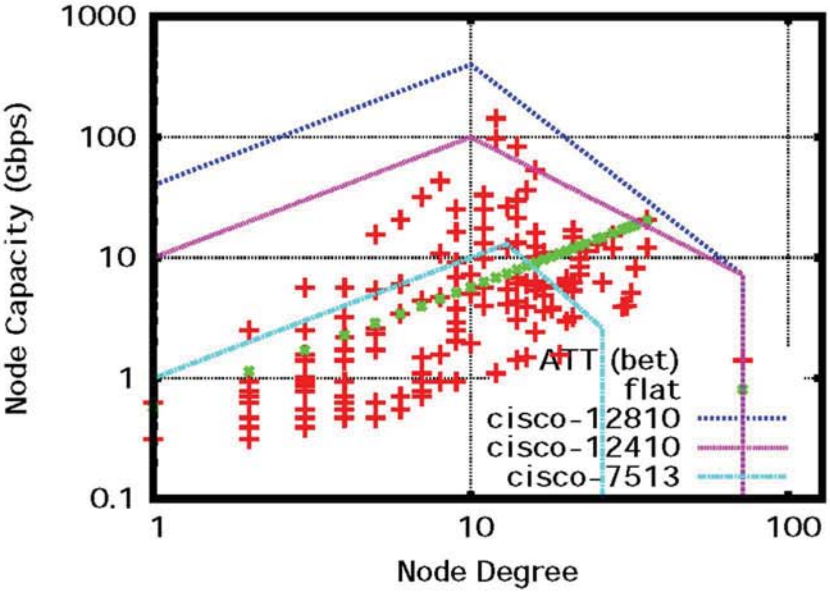
We prepare the link capacity distribution for E edges by following procedures. First, the largest link capacity B_0 is assumed to be 40 Gbps, and the number of C_0 network interfaces that support B_0 is prepared. Then, the second largest link capacity B_1 and the number C_1 is defined by, $B_1 = B_0 / \beta$ and $C_1 = \alpha \times C_0$. More formally, we set,

$$B_i = B_{i-1} / \beta,$$

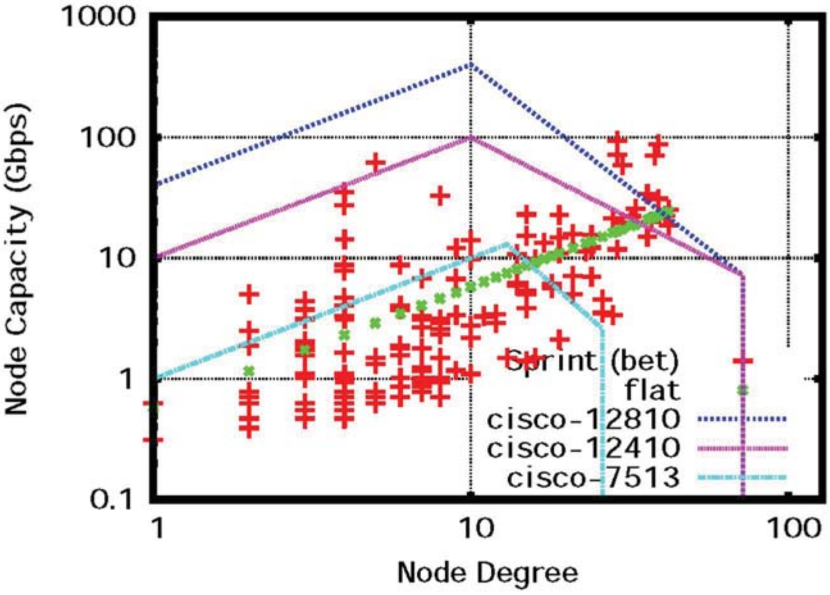
and,

$$C_i = \alpha \cdot C_{i-1}.$$

We apply these operations until $\sum C_i$ is greater than the number of edges E. Note that when we set α and β to be the same value, the link capacity distribution follows power-law with an exponent -1. Figure 8 presents the link capacity distribution by setting B_0 as 40 Gbps, C_0= 4, α=β= 2. We then assign the link capacities prepared by the above procedures to each link according to the edge betweenness centrality such that higher link capacity is assigned to a link having higher edge betweenness centrality.

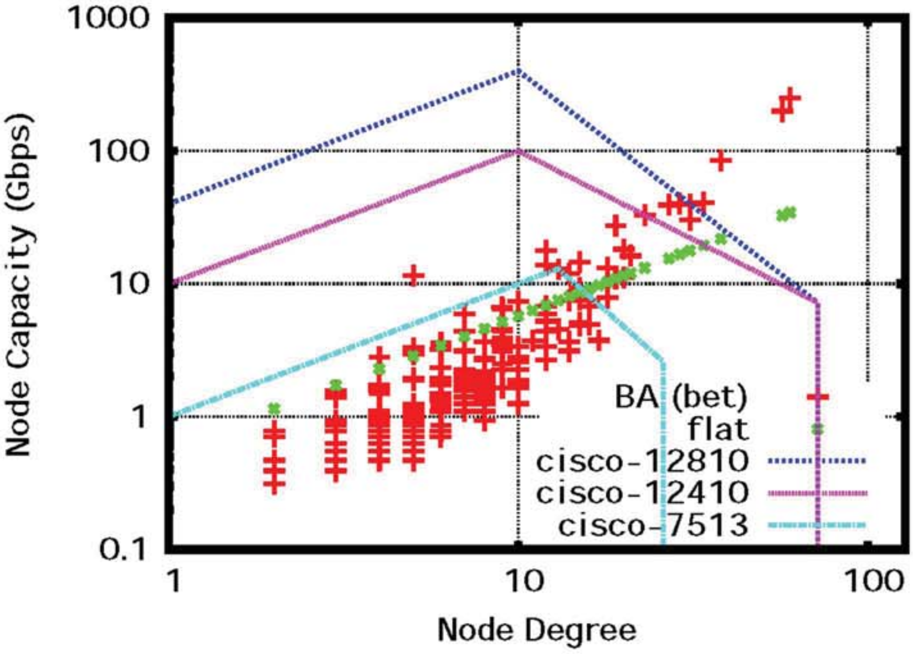


(a) AT&T topology

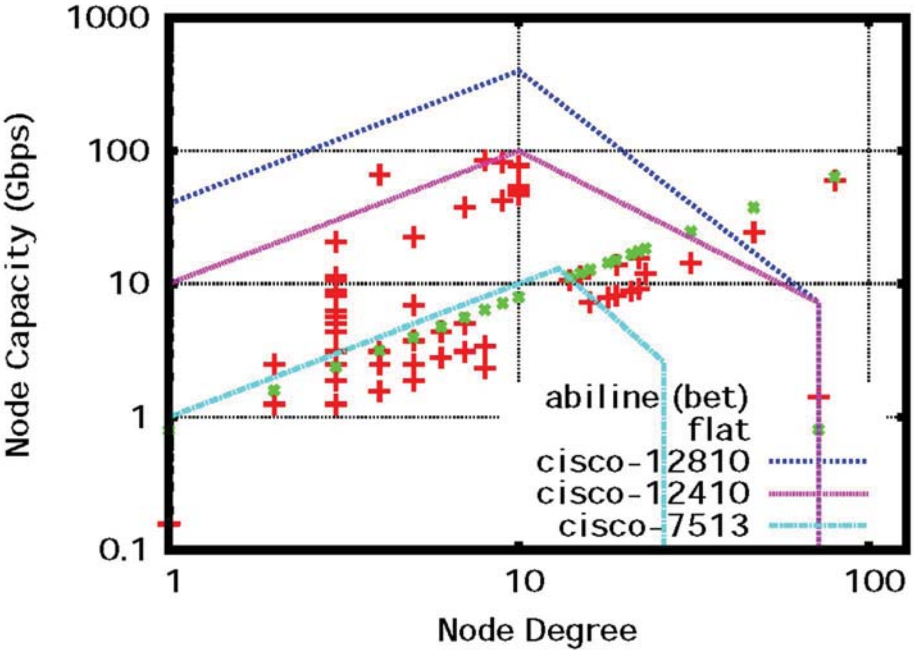


(b) Sprint topology

Figure 9. The required processing capacity of routers in ISP topologies



(a) BA topology



(b) Abiline topology

Figure 10. The required processing capacity of routers in model-based topologies

Figure 9 and Figure 10 show required processing capacities of routers for each topology. The required processing capacity of a router is defined as the total amount of link capacities attached to the router. In each of the figure, horizontal axis is degree of routers, and the vertical axis is the required processing capacity of routers. We show the required link capacity when we assign the link capacity according to the procedures described above. For reference purpose, we also show the results when we assign identical bandwidth to links keeping the same amount of total link capacity.

Looking at the results of Abilene topology, we observe that routers around 10 degree require more processing capacity than other nodes, which agree with the discussion in Ref. [3]. Most of required processing capacity of routers is within the boundary of the current router's technology. Note here that the observation here is the same to the discussions in Ref. [3]. However its meaning is different; our results are caused by the power-law link capacity distribution, while results of Ref. [3] are obtained by the maximizing the throughput in the topology.

Comparing with the results of ISP's router-level topologies and the result of BA topology, we can see the required processing capacities of routers are feasible in the case of ISP's router-level topologies, where as it is not feasible in the BA topology. In the BA topology, because the link connected to hub nodes has larger edge betweenness centrality, the hub node must attach network-interface cards that support larger link capacity. As a result, the required processing capacity of routers increases as their degree increases.

These results are obtained by assuming the wire-speed architecture of routers, and do not use an actual traffic between nodes. The simple design rules of link capacities make the link capacity distribution to follow a power-law with exponent -1, which in turn accommodates the amount of flow (edge betweenness centrality) having power-law attribute with exponent -1 efficiently. Thus, it seems that the power-law relationship of the link capacity is essential to characterize ISP networks. We next validate this observation by using disclosed information of link capacities of the IIJ network that is one of commercial ISPs in Japan.

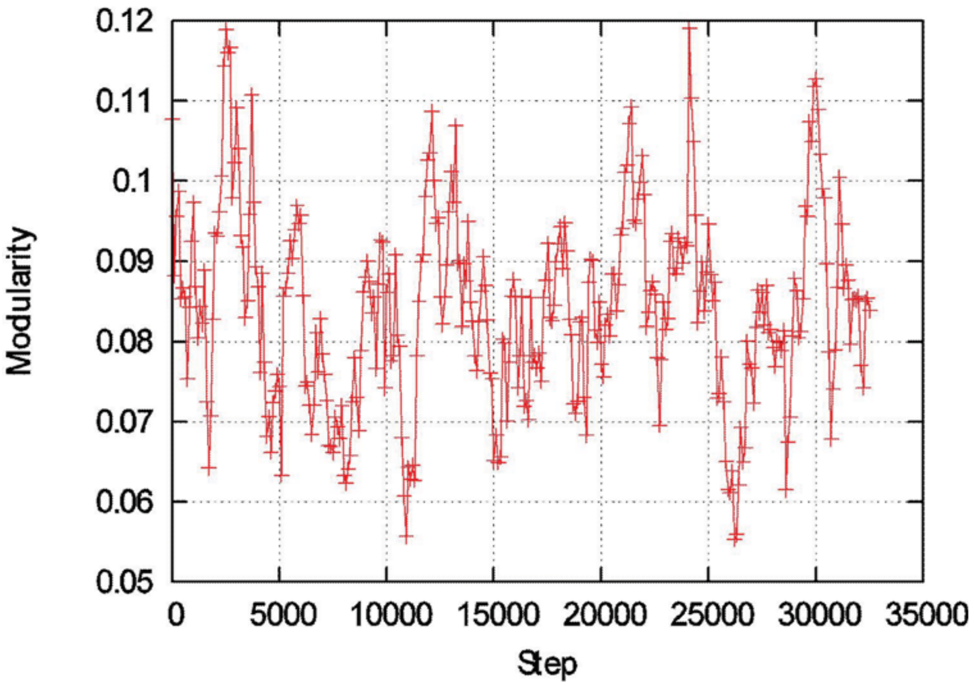


Figure 11. Changes in the modularity

4.3. Relation between structural properties and flow distribution

Results of previous section show that the flow distribution of ISP topologies has a power-law attribute. That is, there exist a few links that accommodate lots of flows and many with a few number of flows. In this section, we explore the reason why the flow distribution exhibits the power-law.

As the first step, we investigate the origin of the modularity structure in ISP topologies. Ref. [3] reveals that high-degree nodes tend to connect with low degree nodes due to the technical constraints of the commercial router, and the topologies are characterized by $d2$-level information. More precisely, the network throughput is characterized by the sum of the product of degree for two connecting nodes. Hereafter, we introduce the term $s(g)$ that represent the sum of the product of degree for two connecting nodes for a graph $g(V\ ,E)$ defined by,

$$s(g) = \sum_{ij \in E} d_i \cdot d_j,$$

where E represents the set of edges and d_i is the degree of node i. We first show that both $s(g)$ and the modularity characterizes the topological structure of the ISP topologies. We change the structure of the BA topology through the rewiring such that

$s(g)$ is minimized while keeping $d1$-level information. $d2$-randomizing in [17] is used, but modified to accept the rewiring only when $s(g)$ decreases. The result is shown in Figure 11 where x-axis represents the rewiring step and y-axis the modularity. We observe that the modularity fluctuate even when $s(g)$ decreases, which indicate that the high modularity structure is essential and independent from $s(g)$ to characterize the topological structure of ISP topologies. We therefore again apply the rewiring procedure to the BA topology such that the modularity of the topology increases (See Figure 12 for the results). Here, we introduce the capacity ratio, $c(g)$, defined by the following equation:

$$c(g) = \frac{F(g)}{T(g)},$$

where $F(g)$ is the sum of 2 σ quantile cutoffs on each link, and $T(g)$ is the sum of amount of flow on each link. $c(g)$ accounts for the flow fluctuation on link and is normalized by the total amount of flow. The lower $c(g)$ means that the ISP network requires less amount of link capacity to be tolerable against flow fluctuation. Looking at Figure 13, we observe that as the rewiring progress, i.e., the modularity increases, amount of the additional link capacity decreases. For calculating $c(g)$, we assume that each flow fluctuate with Normal distribution $N(1,1)$. Figure 14 where $c(g)$ is depicted dependent on the modularity also show the strong correlation between the modularity and $c(g)$.

Figure 15 show the relation of total traffic that the topology accommodates under the router's technical constraint. In the figure, x-axis represents the total traffic and y-axis represents $c(g)$. Each point in the figure represents the result of each topology, and we show the results of 300 topologies out of 30,000 topologies obtained through the rewiring process. We first observe that as the modularity increases the total throughput decreases in general. We can also observe that there exists a tradeoff relationship between the modularity and total traffic. However, there also exists the topology that has a higher total traffic and lower capacity ratio. Note that Ref. [7] observes that the power-law degree distribution can result from trade-offs between "distance" metrics. The authors mentioned that the power-law could result from trade-off between various aspects of performance that must be optimized simultaneously. We therefore investigate the origin of the power-law flow distribution by optimizing both the modularity and total traffic. More precisely, in the rewiring process, we accept the rewiring only when the modularity increases and $s(g)$ decreases. The result after 200,000 rewiring steps is shown in Figure 16. The y-axis is the amount of flow that passes through the link and x-axis is the rank of it. The result without rewiring (the original BA topology) and with 200,000 rewiring are plotted. As we expected, the power-law flow distribution is observed through the optimization procedure for modularity and total throughput.

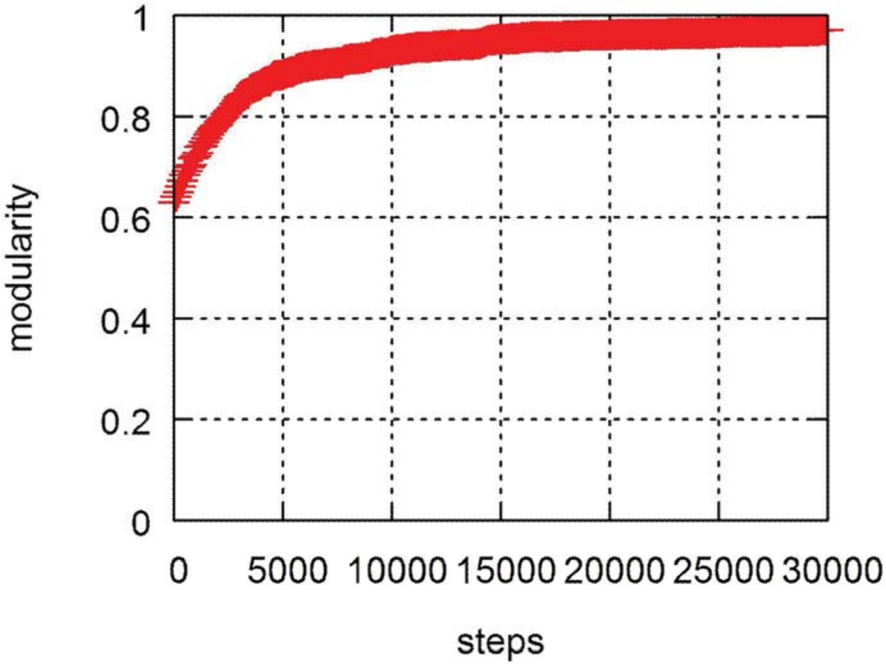


Figure 12. Modularity dependent on the rewiring step

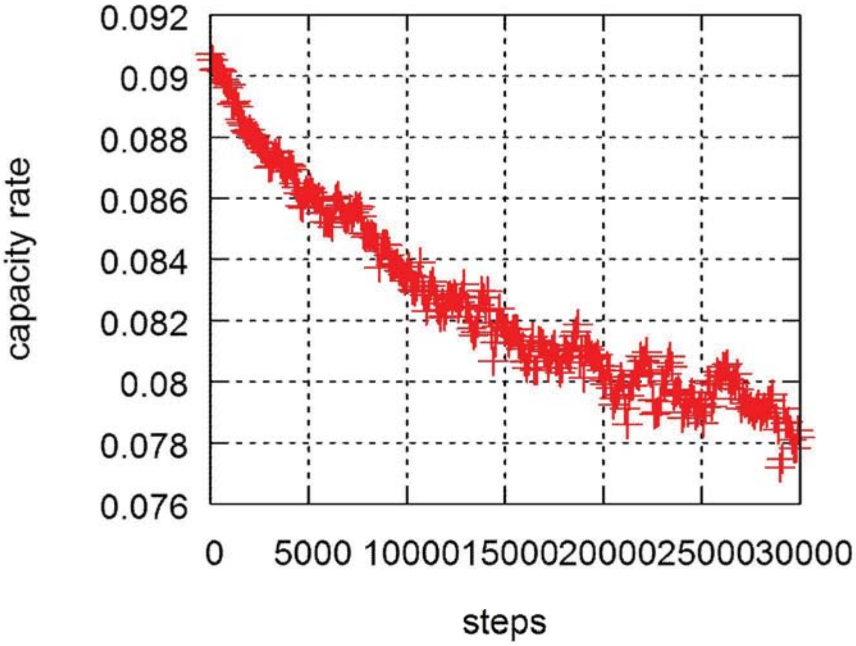


Figure 13. Capacity rate dependent on the rewiring step

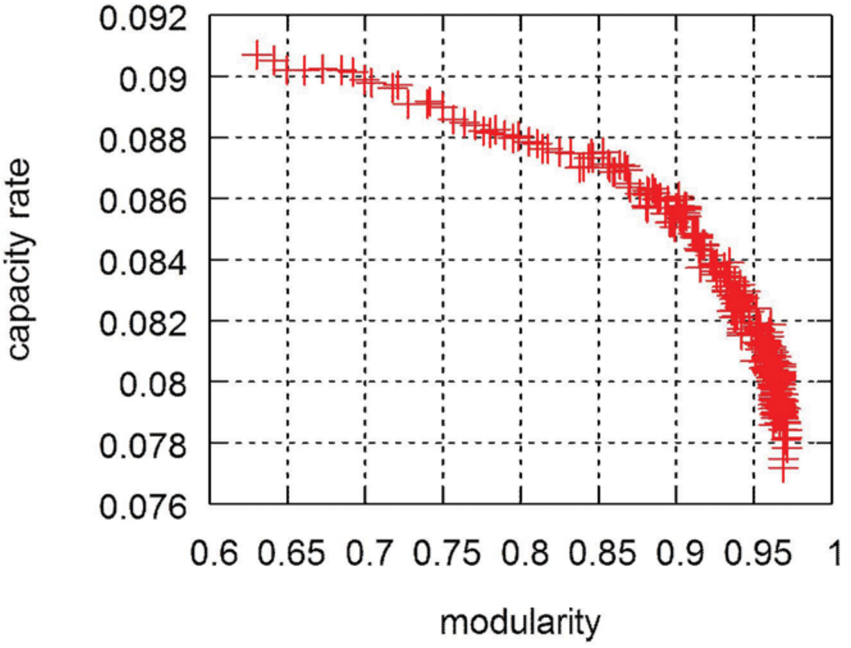


Figure 14. The relation between modularity and capacity rate

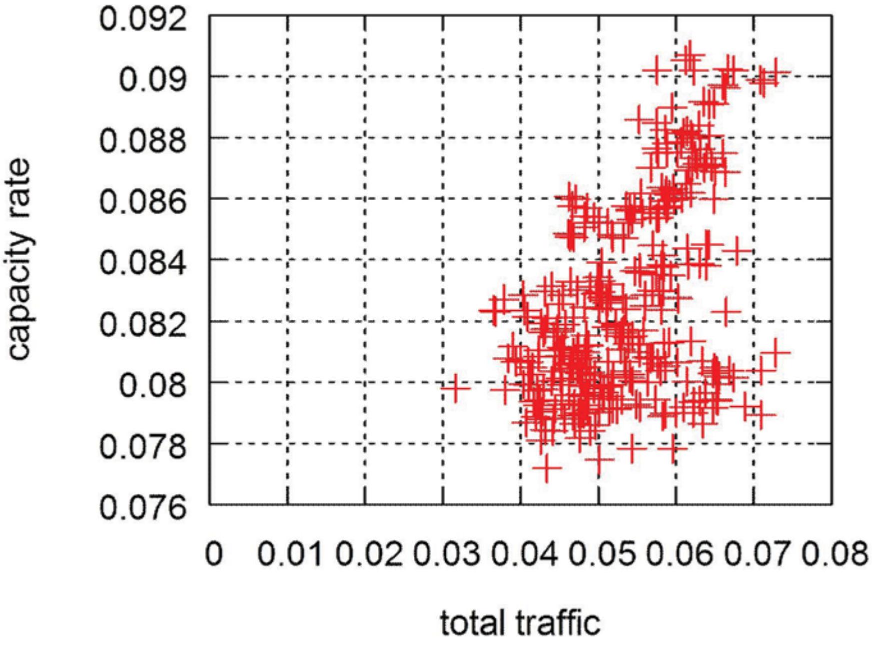


Figure 15. The relation between total traffic and the capacity rate

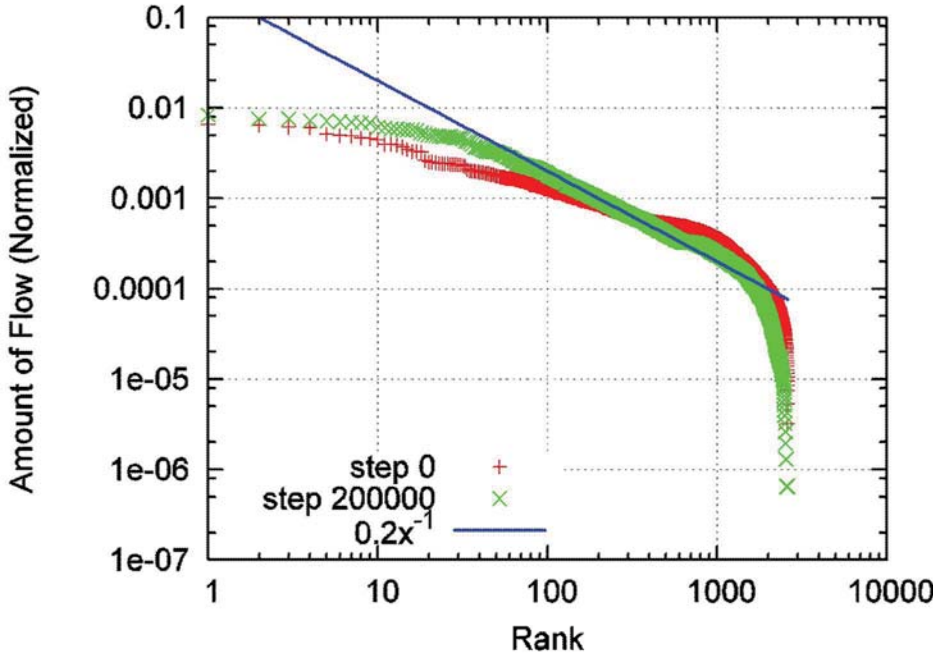


Figure 16. Power-law flow distribution as the result of optimization

5. Conclusion

In this thesis, we focused on the link capacity as a design parameter particular to realistic ISP topologies, and show that both the link capacity in the ISP's backbone network in Japan and the flow distribution obeys power-law. We then explored the reason why the flow distribution exhibits the power-law. Our results indicated that the high modularity structure of the route-level topologies is one of essential topological structures that characterize the performance of router-level topologies. Simulation results showed that the origin of the modularity structure comes from minimizing the flow fluctuation on links. We then investigated the origin of power-law flow distribution in ISP topologies, and showed that power laws can result from trade-offs between network throughput and the amount of additional link capacity to be tolerable against flow fluctuation.

In this thesis, we focus on the amount of flow on links. However, the flow distribution of node is also an important factor to characterize the performance of the communication network because of the router's technological constraints. One of our future work is to investigate the flow distribution on nodes and consider the deployment and operational cost of routers.

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References

[1] S. Floyd and V. Paxson, "Difficulties in simulating the Internet," *IEEE/ACM Transactions on Networking*, vol. 9, pp. 392-403, Feb. 2001.

[2] M. Faloutsos, F. Faloutsos, and C. Faloutsos, "On power-law relationships of the Internet," *in Proceedings of ACM SIGCOMM*, pp. 251-262, Oct. 1999.

[3] L. Li, D. Alderson, W. Willinger., and J. Doyle, "A first-principles approach to understanding the Internet's router-level topology," *ACM SIGCOMM Computer Communication Review,*, vol. 34, pp. 3-14, Oct. 2004.

[4] S. Hosoki, S. Arakawa, and M. Murata, "A model of link capacities in ISP's router-level topology," in *Proceedings of International Conference on Autonomic and Autonomous Systems (ICAS 2010)*, March 2010.

[5] R. Fukumoto, S. Arakawa, T. Takine, and M. Murata, "Analyzing and modeling route-level Internet topology," *in Proceedings of ICOIN 2007*, Jan. 2007.

[6] O. Heckmann, M. Piringer, J. Schmitt, and R. Steinmetz, "Generating realistic ISP-level network topologies," *IEEE Communications Letters*, vol. 7, Jul. 2003.

[7] A. Fabrikant, E. Koutsoupias, and C. Papadimitriou, "Heuristically optimized trade-offs: A new paradigm for power law in the Internet," *in Proceedings of the 29th International Colloquium on Automata, Languages and Programming*, pp. 110-122, Jul. 2002.

[8] N. Berger, B. Bollobas, C. Borgs, J. Chayes, and O. Riordan, "Degree distribution of the FKP network model," *in Proceedings of International Colloquium on Automata*, pp. 725-738, Jul. 2003.

[9] J. Hamelin and N. Schabanel, "An Internet graph model based on trade-off optimization," *European Physical Journal B*, vol. 38, pp. 231-237, Mar. 2004.

[10] N. Spring, R. Mahajan, D. Wetherall, and T. Anderson, "Measuring ISP topologies with rocketfuel," *IEEE/ACM Transactions on Networking*, vol. 12, pp. 2-16, Feb. 2004.

[11] A. Barabasi and R. Albert, "Emergence of scaling in random networks," *Science*, vol. 286, pp. 509-512, Oct. 1999.

[12] T. Bu and D. Towsley, "On distinguishing between Internet power law topology generators," *in Proceedings of INFOCOM*, pp. 1587-1596, Jun. 2002.

[13] H. Maruta, *GLOCOM Review*, vol. 8, no. 4, 2003, (Title in Japanease. Available at page 10 of http://www.glocom.ac.jp/top/project/odp/library/76_01.pdf).

[14] Internet Initiative Japan. http://www.iij.ad.jp/.

[15] R. Guimera and L. Amaral, "Functional cartography of complex metabolic

networks," *Nature*, vol. 433, pp. 895-900, Feb. 2005.

[16] N. Newman, "Modularity and community structure in networks," *in Proceedings of the National Academy of Sciences of the United States*, vol. 103, pp. 8577-8582, Apr. 2006.

[17] P. Machadevan, D. Krioukov, K. Fall, and A. Vahdat, "Systematic topology analysis and generation using degree correlation," *in Proceedings of SIGCOMM 2006*, Aug. 2006.