Analysis of Network Heterogeneity by Using Entropy of the Remaining Degree Distribution

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Presentation Outline
1. Background and objective
2. Explain the measurement
3. Router-level topologies calculated by the measurement
4. Describing some topological characteristics by changing the value of the measurement through a rewiring process
5. Conclusion and future work

Backgrounds
- Designing the Internet that has adaptability and sustainability against environmental changes is important
  - Adaptability against the failure of network equipment
  - Sustainability against changes of traffic demand

- One of the key properties to focus on is the network heterogeneity
  - “Complex networks display heterogeneous structures from different mechanisms of evolution" [2]

Goal & Objective
- Goal
  - To design networks that has adaptability and sustainability focusing on the network heterogeneity

- Objective in this work
  - Confirming mutual information is usable to evaluate the network heterogeneity of topological structure of router-level topologies

Mutual information and Network heterogeneity
- Mutual information
  - The amount of information that can obtain about one random variable X by observing another variable Y
  - \( I = H(X) - H(X|Y) \)
    - \( H(X) \): Entropy, \( H(X|Y) \): Conditional entropy

- Diversity of a topology can be measured
  - \( Y \): a part of the topology
  - \( X \): the rest part of the topology

- Mutual information is high -> Less diverse
  - Much information can obtain about X by observing Y

- Mutual information is low -> Diverse
  - A little information can obtain about X by observing Y

Remaining degree distribution as the random variable
- Solé et al. [2] studied complex networks by using remaining degree distribution as the random variable
- Focus on the relationship of pairs of nodes connected to each other
  - Relationship: degree pattern of those two connected nodes
    - (Number of links connected to a node)

- \( Y \): degree of a node connected to a randomly selected link
- \( X \): degree of a node connected to the other end of that link

- Mutual information is high -> Less diverse
  - Much information can obtain about X (the degree of a node which connected to one side of a link) by observing Y (the degree of a node connected to the other side of the link)

- Mutual information is low -> Diverse
  - A little information can obtain about X by observing Y
Solé et al. calculated mutual information of some complex networks
• Showing even though $I$ is almost the same, $H(X)$ and $H_i(X(Y))$ is different in some case

$I = H(X) - H_i(X|Y)$

$H$: Entropy
• High when Degree distribution become biased, and gets close to power

$\text{Average hops of topologies obtained by setting} H, H_c \text{ as } H = H_c \text{ from 1 to 5}$
• $U(G)$ converge to approximately zero
• When $H$ increases higher than 3, the average hop distance decreases

$\text{Because router-level topologies obey power-law around} H = 4$
Mutual Information and the Characteristic of Topologies

- Generating topologies having different $H_c$, but having the same degree distribution, and compared their diversity
  - Topologies having the same degree distribution has the same $H$
  - under the same $H$, changing $H_c$ is equal to changing $I (I = H - H_c)$

- Generating topology has pre-specified $I$
  - Minimizing the potential function $U_I(G)$ by simulated annealing
    - $B'(G) = |I - I(G)|$
    - $I(G)$ is calculated by the topology $G$ generated in the optimizing search process
  - Initial topology
    - Obtained by BA model (same number of nodes and links with AT&T)
  - Changing method
    - Random rewiring that leaves the degree distribution unchanged[14]

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Conclusion and Future Work

- Conclusion
  - Investigating the network heterogeneity of router-level topologies by using mutual information
    - Router-level topologies have higher mutual information than model-based topologies
  - Generating topologies with different mutual information
    - When the distribution is the same
      - Topology has regularity when mutual information is low

- Future work
  - Evaluate network performance of topologies with different mutual information
  - Apply this measure to designing information network that has adaptability and sustainability against environment changes