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Improving Reliability of Inter-connected Networks through Connecting Structure

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Research background

- **The Internet is widely spreading**
  - It plays an important role in our life as social infrastructure
  - Reliability is called for when failures occur
- **Improving Reliability is important**
  - There are many previous work to enhance reliability
  - However, most of them intend for constructing a single network
  - In fact, the Internet consists of many small networks which are mutually connected.

Reliability of inter-connected network should be considered too

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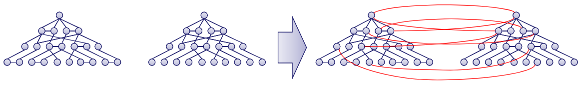
Research purpose and approach

We explore a better structure of inter-connected network in terms of reliability

- **Approach**
  1. We prepare two local networks which are identical
  2. We construct various inter-connected networks
     - Set the layer based on hop count from the largest degree node
     - Connect among local networks by layer
  3. Evaluate reliability against failures



two local networks　　inter-connected network

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Evaluation of reliability of a single network[4]

In Ref. [4], reliability of various structure of networks is evaluated by comparative evaluation

- **Five network types are prepared based on connecting probability $P(i,j)$ on each node pairs (i,j)**

$$P(i,j) \propto e^{-\frac{D_{ij}}{\lambda}} \cdot e^{-\frac{x_{ij}}{\zeta}}$$

  - D_{ij} : The depth of their nearest common ancestor a_{ij}
  - x_{ij} : The distance between nodes i and j ($x_{ij} = (d_i^2 + d_j^2)^{\frac{1}{2}}$)
- **Evaluate robustness of each network type**
  - Metrics
    - Congestion Robustness : the maximum load imposed on node
    - Connectivity Robustness : the size of the largest connected component
  - Result
    - Structure with $\lambda = 0.5, \zeta = 0.5$ is the highest reliable

[4] P. S. Dodds, D. J.Watts, and C. F.Sabel, "Information exchange and the robustness of organizational networks," in *Proceedings of the National Academy of Sciences (PNAS)*, vol. 100, Oct. 2003, pp. 12516–12521.

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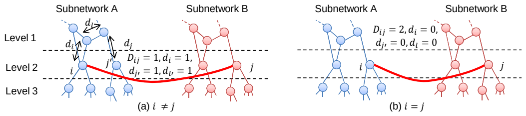
Network construction algorithm

1. Calculate connecting probability on each node pairs

$$P(i,j) \propto e^{-\frac{D_{ij}}{\lambda}} \cdot e^{-\frac{x_{ij}}{\zeta}}$$

- D_{ij} : The highest layer on the shortest path between (i,j')
- x_{ij} : The distance between (i,j') ($x_{ij} = (d_i^2 + d_j^2 + d_l^2)^{\frac{1}{2}}$)　Extended for [4]
- j' : The node corresponding to the node j in the network with node i

2. Choose l node pairs based on $P(i,j)$ and connect



Example of connecting between node i in 2nd layer and node j in 2nd layer

[4] P. S. Dodds, D. J.Watts, and C. F.Sabel, "Information exchange and the robustness of organizational networks," in *Proceedings of the National Academy of Sciences (PNAS)*, vol. 100, Oct. 2003, pp. 12516–12521.

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Parameters for constructing various inter-connected networks

- **Five types of inter-connected networks can be constructed by setting the parameters, (λ, ζ)**
  - If λ is small, nodes with upper level tend to connect
  - If ζ is small, near nodes tend to connect

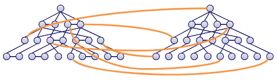
λ	ζ	Connecting structure
∞	∞	Random (R)
∞	0.05	Local Team (LT)
0.05	∞	Random Interdivisional (RID)
0.05	0.05	Core-periphery (CP)
0.1~0.9	0.1~0.9	Multiscale (MS)

Extended for [4]

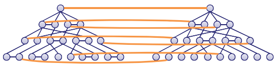
[4] P. S. Dodds, D. J.Watts, and C. F.Sabel, "Information exchange and the robustness of organizational networks," in *Proceedings of the National Academy of Sciences (PNAS)*, vol. 100, Oct. 2003, pp. 12516–12521.

Connecting structure (1/2)

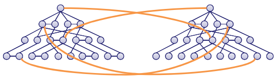
- **Random (R)**
  - Links are added randomly
- **Local Team (LT)**
  - More links are added between nodes in the same layer
- **Random Interdivisional (RID)**
  - More links are added between nodes that have long distance



Random



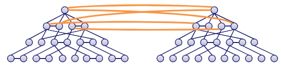
Local Team



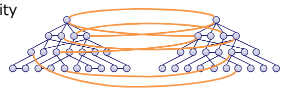
Random Interdivisional

Connecting structure (2/2)

- **Core-periphery (CP)**
  - Links are added between near and top nodes
  - Get the topology with densely connected "core" and sparsely connected "edge"
- **Multiscale (MS)**
  - Made by setting intermediate values of parameters
  - Get the topology that the link density decreases as the hierarchical level decreases



Core-periphery



Multiscale

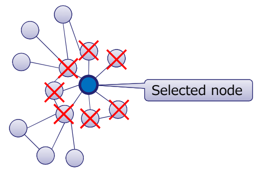
Evaluation environment

- **Use BA topology[9] as a local network**
  1. Prepare a complete graph with m_0 nodes
  2. Repeat following steps until the number of nodes equal to n
     a. Set a new node
     b. Select m ($\leq m_0$) nodes with preferential probability $\frac{k_i}{\Sigma_j k_j}$ (k_i is the degree of node i)
     c. Add links between a new node and selected nodes
- **Set following values when constructing inter-connected network**
  - Number of nodes in a local network : 500, 1000
  - Average degree : 2, 3
  - Number of inter-connected links : 50, 100, 200

[9] A. Barab´asi and R. Albert, "Emergence of scaling in random networks," Science, vol. 286, pp. 509–512, Oct. 1999.

Failure models

- **Multiple failures**
  - A node fails at random one by one
- **Disaster failures**
  - Following nodes fail occurs simultaneously
    - A node selected randomly or with priority to large degree
    - Its neighbor nodes



Metrics to evaluate the reliability

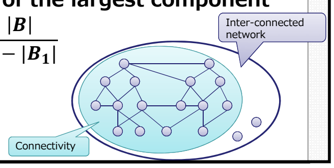
- **Average hop length H : the average hop length for all node pairs**

$$H = \frac{\Sigma_{i \in B} \Sigma_{j \neq i, j \in B} d_{ij}}{|B|(|B| - 1)}$$

  - N : The number of nodes
  - B : A set of nodes in the largest connected component after the failures occur
  - d_{ij} : The shortest path length from node i to node j
- **Connectivity C : the ratio of the largest component**

$$C = \frac{|B|}{N - |B_1|}$$

  - B_1 : The set of failed nodes



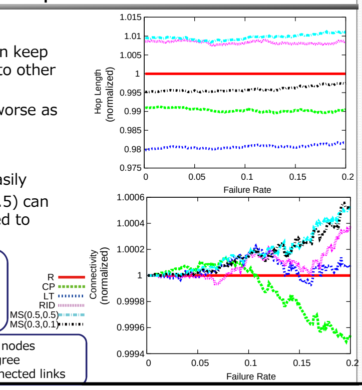
Reliability against multiple failures

- **Average hop length**
  - CP, LT and MS(0.3,0.1) can keep hop length low compared to other structures
  - LT's values is sometimes worse as well as R's values
- **Connectivity**
  - CP loses its connectivity easily
  - MS(0.3,0.1) and MS(0.5,0.5) can keep connectivity compared to other structures

R : links existed at random
CP : densely connected "core"
LT : connecting nodes in the same layer
RID : links with long distance
MS : intermediate structure

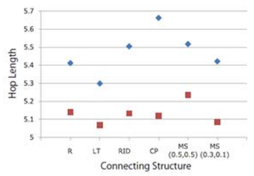
(above) 500 nodes
2 average degree
50 inter-connected links

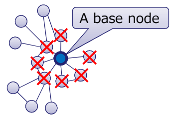
(below) 1000 nodes
3 average degree
100 inter-connected links



Reliability against disaster failures

- **Select a base node at random (■)**
  - CP, LT and MS(0.3,0.1) can keep the average hop length low
- **Select a core node as a base node (◆)**
  - CP cannot keep the value





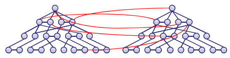
R : links existed at random
CP : densely connected "core"
LT : connecting nodes in the same layer
RID : links with long distance
MS : intermediate structure

1000 nodes
2 average degree
50 inter-connected links

Conclusion and future work

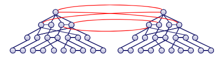
- **Conclusion**
  - Evaluate reliability against failures of networks having different structure
  - Keep high reliability using various nodes, not only using central nodes in network

Reliable structure



with links throughout network

Not reliable structure



with upper links densely

- **Future work**
  - Investigate reliability when local networks have different topology
  - Evaluate with focusing on capacity or flow on each link in network