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Title

Analysis and Strategies for Improving Robustness and Efficiency of Interconnected Networks

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Abstract

Interconnected modular networks have been observed in many complex systems in biology, society, science and technology, as well as the Internet. Among those types of complex systems, the Internet is rapidly developing toward the next generation of the Internet of Things (IoT), which accelerates the emergence of interconnected modular architectures even further. However, the best way how to design such interconnected networks, which can meet various changes in environment and service demands, remains an important issue that has not been addressed yet. When providing Internet services, the *Network of Networks* (NoN) architecture should not spread malicious information or fall into the state of cascading failures, while on the other hand, it should pass urgent and important legitimate information to the network. In this thesis, we propose an NoN model inspired by the nature of interconnected modular networks in the brain. Even though our proposed NoN model can prevent malicious information from diffusing from inside of a subnetwork to another, it cannot prevent diffusion starting from interconnecting links. Therefore, interconnecting links still have potential to serve as sources of both legitimate and malicious information. In order to find a strategy for changing the speed of information diffusion, we establish a method for configuring the connectivity within and between subnetworks of the interconnected networks from the viewpoint of nodal influence and its correlations, i.e., assortativity. To achieve this goal, we simulate information diffusion and investigate the relationship between connectivity and diffusion speed. We confirm through simulation experiments that our proposed model can diffuse information as fast as a purely interconnected networks, which do not prevent any information on the interconnecting links, when we configure highly influential nodes within subnetworks and interconnect them assortatively. The results also show that our proposed model reduces the speed of information diffusion to the nearly the slowest case of an independent subnetwork without any interconnecting links, when we configure stretched subnetworks and disassortatively select non-influential nodes as endpoints of interconnecting links.

Keywords

Internet of Things

Network of Networks

Brain Networks

Centrality

Assortativity

Contents

List of Figures

List of Tables

1 Introduction

Interconnected modular networks, often referred to as *Network of Networks* (NoN), have been observed in many complex systems in biology, society, science and technology, for exampe the Internet [2–7]. Among those types of complex systems, the Internet is rapidly developing toward its next generation as the Internet of Things (IoT), which permits connecting various kinds of interconnected devices in everyday life via the Internet protocol and is expected to accelerate the emergence of modular architectures even further.

An example of such modular architectures in the future Internet is the functionally interconnected networks in smart cities [8]. In the future IoT society, the number of devices connected to the Internet and the type of services provided through the Internet are expected to show an explosive and continuous increase. Smart cities automatically collect data from IoT devices and intelligently integrate them for improving services for healthcare, surveillance, infrastructure, public utilities, etc., resulting in the realization of smart homes, smart grid, and more. Simple examples in smart homes are air conditioning systems that capture temperature, humidity, and air circulation from IoT devices and provide best services responding to a variety of situations. Another example is the collaboration of a number of motion sensors embedded in IoT devices, that will be useful for health-care of elderly people or anomaly detection in security surveillance. In a more macro-scale situation like smart grids, conventional standalone metering systems in basic infrastructures, such as electricity, gas, or water will be integrated into a more intelligent and automated system that can conserve energy resources. In these situations of smart cities, a processing halt in one service module stops the functions in other interdependent modules. Adding to the situations we can predict at the moment, the number of such automated and independent service systems over the IoT infrastructure is expected to increase in future smart cities.

Although these new interconnected Internet services are currently appearing, the way to design an NoN architecture that can meet various changes in environment (e.g. disasters or epidemics) and service demands (e.g. availability, communication efficiency, or cost) remains an important issue that has not been addressed yet. When providing Internet services, the NoN architecture should not spread malicious information and fall into the state of cascade failures, while on the other hand it should pass urgent and important legitimate information to the network. Regarding the design of the NoN architecture, the procedure can be divided into two parts: *controlling information flow* and *configuring topological connectivity*.

Therefore, we first focus on systematic interdependent models of NoN for controlling information flow, which is a highly discussed topic in the research field of modular interconnected networks and has been previously discussed [1, 9]. A conventional and well-known NoN model is based on a real-world cascading failure between power grid network and supervisory control networks that took place in Italy, 2003 [9]. In this case, the heavy interdependence between the two networks caused a small fraction of failures in one of the networks to increase to a large scale cascade in both networks, resulting in a massive blackout in the major part of Italy. However, many biological systems have in fact high robustness against network failures. Morone *et al.* [1] proposed another NoN model from the perspective of neuroscience, i.e., with respect to networks in the brain. This NoN model, termed as *Brain NoN* hereafter, incorporates the characteristics of activation rules of neural firings in brain networks that are well-known for their high robustness [10, 11]. Because of the only local interdependencies of each network, node failures in Brain NoN affect only nodes of the same network, in contrast to the power grid NoN model. This robustness against cascade failures on interdependent Brain NoN can also be applied to the interconnection of information networking services. Application of such activation rule from the Brain NoN to services in information networking, however, has not been considered so far, and in this thesis we reinterpret this rule so that it can match the situations we mentioned above.

Second, regarding topological connectivity, there are still two important questions to be answered in order to satisfy the environmental changes and service demands in those interconnected information networks: (i) *how is the connectivity within modules?* and (ii) *how is the connectivity between modules?* In this thesis, we attempt to answer those questions from the viewpoint of *influential nodes* and *its correlations*. Influential nodes in networks are defined as those nodes that have a large influence on acceleration or suppression of diffusion, and they have been investigated in many complex systems [12–22] as the control of the influence from a tiny fraction of nodes over the whole network. Node correlation [23, 24] is formulated based on the correlation of node degrees of two nodes and termed as *assortativity*. In this work, we expand the definition of assortativity so as to measure the correlation of any kind of nodal influence.

The aim of our work is to design an NoN architecture for information networking services that meets environmental changes and service demands of each service module, which can be summarized as high robustness and communication efficiency. The ideal NoN architecture should prevent malicious information from spreading out, while it should diffuse important information as quickly as possible. It should be noted that the term “robustness” is used from software viewpoint, not hardware. For this aim, we first propose an NoN model inspired by the Brain NoN that matches situations in information networking with service interdependence, which never leads to cascading failures. Second, by taking the nodal influence and its correlation into account, we propose a method to configure the intra- and inter-modular connectivities and evaluate the performance of our proposed NoN. Evaluation results reveal that this NoN model can realize both ideally fast and slow speed of information diffusion by changing its topological connectivity, and thus it can achieve robustness and efficient communication, unlike the conventional NoN model without control on interconnectivity.

2 Related Work

In this section, we introduce existing NoN models [9, 25] and explain the mechanism of Brain NoN [25]. Furthermore, we provide the definition of nodal influence and nodal assortativity, which are basic ideas of our proposed method to design interconnected NoN with robustness and communication efficiency.

2.1 Models of Network of Networks

With the emergence of the interconnected modular architecture in today's networks, several NoN models have been proposed [9, 25] to reproduce this interdependency between interconnected networks. A prominent example in this field is the interconnected architecture of a power-grid network and its supervising network [9]. The two networks mutually depend on each other and cannot be operationed separately: the power-grid depends on the supervising network for its control, while the supervising network depends on the power-grid for electrical supply. Thus, a small network failure can result in a cascading failure over a wide range of the network, as seen in a real-world massive blackout in Italy, 2003 [26].

By contrast with this fragile NoN model, another NoN model, called Brain NoN, was proposed to get inspiration from the modular architecture in the mammalian brain network [25]. The brain network is composed of a number of modules, which have different functions respectively, and intra- and inter-modular links play different roles in the processing. Intra-modular links simply transfer information among nodes (i.e. neurons) within the same module, whereas inter-modular links control the cooperation of different functions in the brain, e.g., vision, recognition, movement, etc.. For these different roles, the former intra-modular links are always on standby while the latter inter-modular links are activated only when the two endpoint nodes are activated. The inter-modular dependency does not affect intra-modular connectivity. The modular structure of the Brain NoN's architecture and its local interdependency makes it more robust than the conventional NoN model of the power grid with its network-wise interdependency, which causes cascading of failures.

In the Brain NoN model, nodes can have three different states: *active, input*, and *no-input*. Each node can be active only when its own and its neighbors' input satisfy a certain condition. These three states of node i are determined by two variables, *input variable* n and *activation*

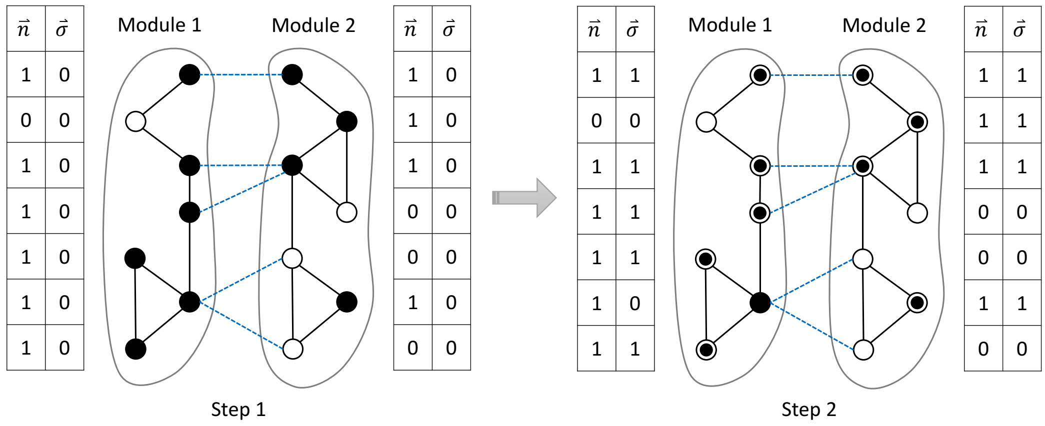


Figure 1: Activation rule in Brain NoN model [1]

variable σ, as follows:

◉ : active ($n_i = 1, \sigma_i = 1$)

● : input ($n_i = 1, \sigma_i = 0$)

○ : no-input ($n_i = 0, \sigma_i = 0$)

The patterns of each circle represent the node states corresponding to Figure 1, which shows an example of state transition in a Network of 2 Networks (2-NoN) of the Brain NoN model. The values for the input variable n are assumed as given and they sequentially determine the values for the activation variables σ. When node i has no inter-modular links, the input signal n is just interpreted as activation state, and the value of σ is defined as

$$\sigma_i = n_i, \quad \text{for } k_i^{out} = 0,$$

where k_i^{out} represents nodal out-degree for inter-modular links of node i.

Then, when node i has one inter-modular link, inputs on both node i and its counterpart node j are required for activating node i, and the value of σ is defined as:

$$\sigma_i = n_i n_j, \quad \text{for } k_i^{out} = 1.$$

Moreover, in the generalized case that node i has multiple inter-modular links, i.e., when $k_i^{out} \geq 2$, node i can be active only when its own input and the input of at least one node in onother modules exists. The value of σ is defined as follows:

$$\sigma_i = n_i \left[1 - \prod_{j \in \mathcal{F}(i)} (1 - n_j) \right], \tag{1}$$

where $\mathcal{F}(i)$ denotes the set of nodes connected to node i via inter-modular links.

2.2 Identification of Influence in Networks

The existence of influential nodes and identifying them are important in many research domains such as computer networks, social networks, infrastructure networks, etc. [12–22], since controlling these nodes is key to the efficient operation of networks. Our study also focuses on the vital nodes in order to control acceleration/suppression of information diffusion in interconnected networks. For instance, strong information diffusers play a prominent role for efficient communication, whereas the existence of the strong diffusers could also causes a quick pandemic in the case of viruses. Identification of a set of nodes that maximizes the influence over a network is known as an NP-hard problem [13], and a great number of heuristic solutions have been proposed so far [19, 20, 22].

We focus here on [18] which proposed the *Collective Influence* (CI) algorithm to identify influential nodes. CI of node i represents its influence on other nodes in the same network centered around node i, and it can be regarded as a kind of centrality metric, like betweenness centrality, pagerank, or k-core. The CI algorithm showed superior performances for the identification of influential nodes to other methods using conventional centrality measurements by finding the smallest set of nodes that totally collapses the connectivity of the networks. In other words, removal of nodes found by the CI algorithm is the fastest way to lead a network to a number of micro-connected components. CI of node i is defined as follows:

$$\begin{aligned} \mathrm{CI}_{l=0}(i) &= (k_i - 1) k_i \\ \mathrm{CI}_{l \geq 1}(i) &= (k_i - 1) \sum_{j \in \partial \mathrm{Ball}(i,l)} (k_j - 1) \end{aligned} \tag{2}$$

where k_i denotes the degree of node i, $\mathrm{Ball}(i, l)$ denotes the set of nodes within l hops centered around node i, and $\partial\mathrm{Ball}(i, l)$ denotes the set of nodes on the edge of $\mathrm{Ball}(i, l)$, see Figure 2. In other words, $\partial\mathrm{Ball}(i, l)$ denotes the set of nodes located exactly l hops away from node i. It has been shown that the algorithm reaches its best performance with parameters $l \in \{3, 4\}$ in a network with 100,000 nodes [18].

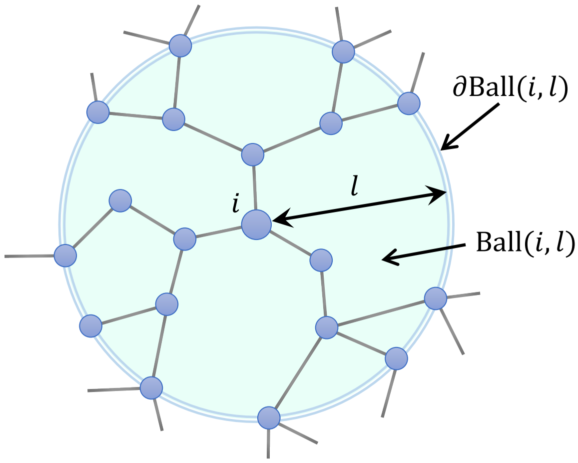


Figure 2: Description of $\mathrm{Ball}(i,l)$ and $\partial\mathrm{Ball}(i,l)$ in Collective Influence algorithm

2.3 Universal Assortativity

Assortativity, i.e., the correlation of nodal degrees, is one of the common characteristics for the evaluation of complex networks proposed by Newman [23]. Furthermore, universal assortativity coefficient was introduced to analyze the assortativity of any part of a network in [24], and it was also used to define the assortativity between networks. In this thesis, we propose a method to construct interconnected networks and change its interconnectivity based on the knowledge of assortativity between networks.

Newman proposed measuring assortativity of a network with the assortativity coefficient [23]. The assortativity coefficient is calculated from the remaining degree distribution $q(k)$ defined as follows:

$$q(k) = \frac{(k+1)p(k+1)}{\sum_j jp(j)}, \tag{3}$$

where $p(k)$ denotes the probability that a randomly selected node has node degree k. The remaining degree of a node in a path corresponds to the number of edges except for the vertex that was arrived along. In other words, the remaining degree of a node in a path corresponds to the node's degree minus 1.

The assortativity coefficient r is defined as follows:

$$r = \frac{1}{\sigma_q^2}\Big(E[(J-U_q)(K-U_q)]\Big), \tag{4}$$

where, and J and K denote variables of the remaining degree, which have the same expected value

$U_q = \sum_j jq(j)$. The term $\sigma_q^2 = \sum_l j^2 q(j) - \left(\sum_k kq(k)\right)^2$ denotes the variance of the remaining degree distribution $q(k)$. Positive and negative values of r indicate an assortative network and a disassortative network, respectively. When r is near zero, nodes are randomly connected with each other independent of their degrees. The range of feasible values of r is based on the degree distribution.

Then, the *universal assortativity coefficient* ρ_l on a link l can be introduced given $q(k)$. This coefficient reflects the contribution of an individual link to the global assortativity coefficient r of the entire network. The definition of the universal assortativity of link l is as follows:

$$\rho_l = \frac{(j - U_q)(k - U_q)}{M\sigma_q^2}, \tag{5}$$

where j and k denote the remaining degrees of the two endpoints of link l, and M denotes the number of edges in the whole network. When $\rho_l > 0$, the link is called an assortative link; otherwise when $\rho_l < 0$, the link is called a disassortative link. A link with $\rho = 0$ has no correlation

3 Information Diffusion Model for Interconnected Networks

Although input to nodes and activation as the result of this input were considered in the Brain NoN model [1], effects of node activation on its neighbor nodes have not been considered. We expand the activation rule of the Brain NoN model to express the communication flow in interconnected networks.

First, we change the interpretation of the node states in the Brain NoN model to states of nodal interfaces (network devices) in information networks NoN, termed as *IN NoN*. The activation of interconnecting links, referred to as *control links* in [1], is coupled with the activation of endpoint nodes of the interconnecting links in the Brain NoN model. For instance, when an interconnecting link is deactivated, both endpoint nodes are also deactivated. This is natural behavior for nodes in the Brain NoN, i.e., neurons, because each neuron cannot control all the connected links separately. In information networks, however, even if one endpoint node is deactivated and thus the interconnecting link is also deactivated, the other endpoint node should maintain its process within the module of the node.

For this reason, the meaning of the states defined by σ in the Brain NoN are re-interpreted as shown in Table. 1, where the activation of nodes is replaced with outer-interfaces. In this context, the input variable n in the Brain NoN represents the input state of information. It should be noted that inner-interfaces are always actively independent of the values of σ or n, so that every node can send or receive information within its module even if outer-interfaces are deactivated. The essential feature of IN NoN is that it never passes unexpected information from one module to another, which is based on the robust feature of the Brain NoN model. Beside IN NoN, we also define Pure NoN in Table 1, a basic model that does not consider the interdependence between modules. IN NoN can control the diffusion speed depending on the connectivity as we show in this thesis, whereas Pure NoN always diffuses at the maximum speed the topological connectivity can produce. This implies that Pure NoN would occasionally not be preferable.

Table 1: Re-interpretation of variables of Brain NoN for information networking

variables	Brain NoN	Pure NoN	IN NoN
$\sigma = 0$	node is inactive	node (outer-interface) is active	outer-interface is inactive
$\sigma = 1$	node is active	node (outer-interface) is active	outer-interface is active

Second, in order to express the flow of information, IN NoN also adopts the notion of time-scale. In this model, the value of variables n and σ at time step t is given by the previous states at time step $t-1$. We then introduce a probability function p_t for nodes to decide whether to have input or not, depending on the states of neighbor nodes. Here, we suppose that each node can pass information at probability δ through active outer- and inner-interfaces whenever they have inputs. Therefore, the probability function $p_t(i)$ for node i is written as follows:

$$p_t(i) = 1 - \prod_{j \in \mathcal{S}(i)} (1 - \delta n_j^{t-1}) \prod_{k \in \mathcal{F}(i)} (1 - \delta \sigma_i^{t-1} \sigma_k^{t-1} n_k^{t-1}), \tag{6}$$

where $\mathcal{S}(i)$ denotes the set of neighbors of node i within the same module, and $\mathcal{F}(i)$ denotes the set of neighbor nodes in the other modules. It should be noted that all inner-interfaces are active, while outer-interfaces are active only when $\sigma = 1$. This equation states that when $\delta = 1$, node i can receive input at time step t if at least one of the neighbor nodes connected via active links has input at the previous time step $t-1$. An important point this equation expresses is that when $\delta < 0$, node i can behave differently depending on the number of input neighbor nodes: the more input neighbors node i has, the more likely node i has input. This expression particularly matches the emergence cases or epidemic cases in information networking, because connectivity of each link is fragile in emergency cases like disasters, and it is not assured that information can reach from a node to neighbor nodes in a short duration.

Then, activation state of node i is rewritten based on the rule in Eq. (1) of the Brain NoN model as follows:

$$\sigma_i^t = n_i^t \left[1 - \prod_{j \in \mathcal{F}(i)} (1 - n_j^t) \right]. \tag{7}$$

Eq. 7 shows that the inter-modular interface of node i becomes active only when node i and at least one neighbor node via inter-modular link has input.

Regarding Pure NoN, the only point differing from IN NoN model is the activation of the outer-interface. The outer-interface in Pure NoN is always active, therefore, the difference appears only on the variable σ, and for Pure NoN it can simply be defined as follows:

$$\sigma_i^t = 1.$$

4 Method for Configuring Connectivity of Interconnected Networks

4.1 Overview

The number of NoN architecture for the Internet services is increasing, and thus we proposed an NoN model that prevents diffusion of malicious information in consideration of interconnectivity in Section 3. However, even though the behavior of information diffusion is tightly related to the topological connectivity, the way to design topologies of interconnected networks that accelerates/suppresses information diffusion has not been investigated so far, particularly under the condition of NoN model like our proposal.

The best strategy for configuring topological connectivity of interconnected networks differs according to the situations: the case of emergency where quick information diffusion is required, and the case of infection where gaining time is important. Furthermore, it is virtually impossible to calculate performance of all combinations of intra- and inter-modular connectivity. Therefore, in our strategy, we configure intra-modular connectivity from the perspectives of node influence identification mentioned in Section 2.2, since node influence and diffusion speed is closely related. Regarding inter-modular connectivity, we propose a method to control the connectivity combining the knowledge of node influence and its correlation.

4.2 Configuring Connectivity within Subnetworks

In order to increase/decrease the power of influential nodes in terms of information diffusion speed in subnetworks, we expand the conventional preferential attachment method and generate topologies with control parameter γ. Given a seed network, we successively add nodes with m links and connect the links to existing nodes. The probability for each link of a new node to be connected to an existing node i is defined as follows:

$$p(i) = \frac{k_i^\gamma}{\sum_j k_j^\gamma}, \tag{8}$$

where k_i denotes the degree of node i. The process finishes when all N nodes are added to the network. Note that ideally preference for Collective Influence $\mathrm{CI}_l(i)$ mentioned in Section 2.2 should be used to for calculating the preference instead of node degree. In this case, however, we start the generative method with small seed networks, while $\mathrm{CI}_l(i)$ works for large scale networks, and thus node degree is adopted here.

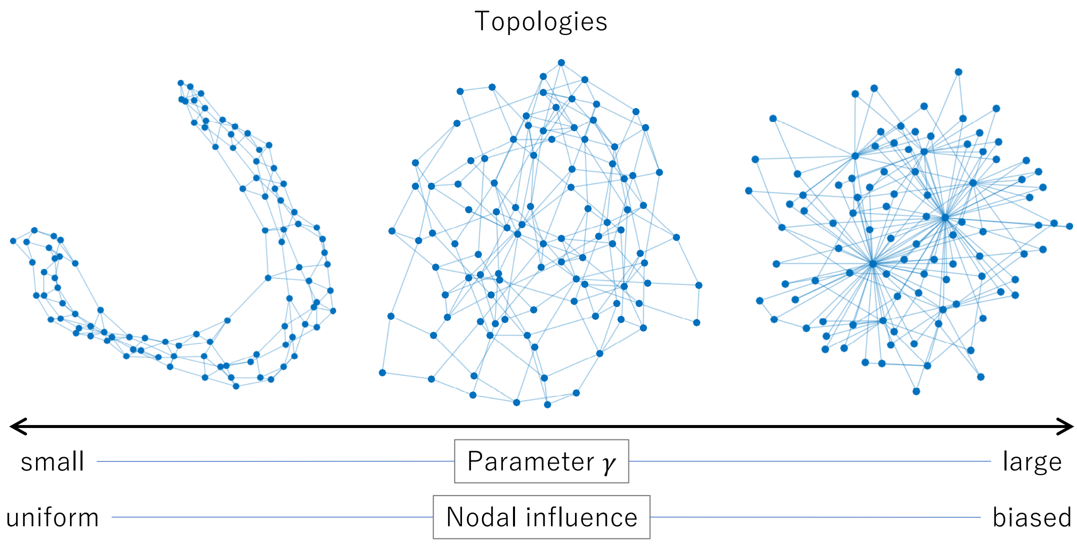


Figure 3: Topologies of subnetworks with various connectivity

Figure 3 shows some examples how the topological shape of subnetworks changes with the parameter γ. When γ decreases, the topology tends to have uniform degree distribution with average degree $\bar{k} \simeq 2m$, and variance of degree approaches zero, thus, also distributing node influence. Whereas when γ increases, more highly influential nodes emerge. As a result, topologies show node degree distribution following power-law $p(k) \sim k^{-\eta}$. This variability in node influence distribution is the reason why we focused on the preferential attachment model.

4.3 Configuring Connectivity between Subnetworks

When generating an interconnecting link between modules that have n nodes respectively, there are n^2 patterns for choices of endpoint nodes of the interconnecting link. Moreover, if we consider multiple interconnecting links, the number of possible pairs of endpoints extends to $O(n^2 \log n)$. On that condition, exploring the best configuration of interconnecting links costs enormous computational cost by a brute force method, and it would be impossible in a large-scale network. Therefore, for the purpose of getting hints on configuring the interconnectivity, we focus on properties of endpoint nodes. When adding an interconnecting link to an NoN, we consider two points: (i) dependency on centrality of both endpoint nodes within each module, and (ii) dependency on correlation of centrality of the two endpoint nodes. All the possible pairs of nodes with a certain centrality value can be expressed by changing these two dependencies respectively, Based on this idea, we investigate which nodes should be preferentially selected as endpoint nodes of interconnecting links for achieving an NoN topology with fast/slow information diffusion. In the following part of this section, we formulate each dependency as *Dependency Coefficient (DC)*. Controlling the two DCs enables a variety of connectivity between modules.

4.3.1 Coefficient for Node Centrality

To begin with, we define the DC of the dependency on centrality itself as DC_{cnt}. Here, we consider the dependency on centrality of each endpoint of interconnecting links independently, and then DC_{cnt} is simply defined as sum of centrality of each endpoint nodes as follows:

$$DC_{cnt}(h, i) = c_h + c_i, \tag{9}$$

where c_h denotes the centrality value of node h within its subnetwork. A high value represents high centrality, and vice versa.

4.3.2 Coefficient for Correlation of Node Centrality

We measure the correlation of node centrality based on the ideas of universal assortativity mentioned in Section 2.3. The universal assortativity is introduced to measure the correlation of node degree centrality between networks as follows.

$$\rho_l = \frac{(j - U_q)(k - U_q)}{M\sigma_q^2}, \tag{5}$$

When calculating universal assortativity ρ_l of an inter-modular link l using Eq. (5), the expected value $U_q = \sum_j jq(j)$ is based on the remaining degree. This is because the universal assortativity just measures the contribution of a link to the entire network; although the universal assortativity can measure assortativity of an interconnecting link between networks, the two networks are regarded as a system, and the interconnecting link is nothing more than a link in a subnetwork. Therefore, the expected value of each endpoint of an interconnecting link must follow the remaining degree of the entire network the link belongs to.

Here, we assume that interconnecting links are generated between two *different* subnetworks independent of the connectivity within each subnetwork. The probability for nodes in each subnetwork to be selected as an endpoint node is the same. When we set $p(c)$ as node centrality distribution of a subnetwork, the expected value of the centrality on an endpoint node of an interconnecting link is also expressed as $p(c)$. Therefore, we define another generalized universal assortativity ρ_l' of an interconnecting link l between network 1 and 2, modifying Eq. (5), as follows

$$\rho_l' = \frac{\left(c_{l_1} - U_{p_1}\right)\left(c_{l_2} - U_{p_1}\right)}{\sigma_{p_1}\sigma_{p_2}}, \tag{10}$$

where c_{l_1} and c_{l_2} denote node centrality of endpoint nodes in network 1 and 2, respectively. U_{p_1} and U_{p_2} denote the expected value of node centrality, defined as $U_p = \sum_j jp(j)$. $\sigma_{p_1}^2$ and $\sigma_{p_2}^2$ denote the variance of node centrality distribution $p(c)$, given as follows $\sigma_p^2 = \sum_l l^2 p(l) - \left(\sum_m mp(m)\right)^2$. We removed the variable M that represented the number of links in the entire network, since this coefficient does not represent contribution of link l to the entire network. Particularly, if networks 1 and 2 have the same node centrality distribution $p(c)$, Eq. (10) can be rewritten as follows:

$$\rho_l' = \frac{\left(c_{l_1} - U_p\right)\left(c_{l_2} - U_p\right)}{\sigma_p^2}. \tag{11}$$

Finally, we define DC_{cor} of the dependency on correlation of node centralities of the two endpoint nodes h and i slightly changing the generalized universal assortativity as

$$DC_{cor}(h,i) = \frac{\left(c_h - U_p\right)\left(c_i - U_p\right)}{\sigma_p^2}. \tag{12}$$

4.3.3 Coefficient for Varying Connectivity between Subnetworks

To configure the connectivity between networks, we consider two aspects as mentioned above: (i) dependency on centrality of both endpoint nodes, and (ii) dependency on correlation of the centrality of the two endpoint nodes. In other words, we combine DC_{cnt} and DC_{cor} into DC and express various interconnectivity between networks. The definition of DC is as follows:

$$DC(h,i) = \left[\frac{DC_{cnt}(h,i) - DC_{cnt}^{min}}{\overline{DC}_{cnt} - DC_{cnt}^{min}} + 1\right]^{r\cos\theta} + \left[\frac{DC_{cor}(h,i) - DC_{cor}^{min}}{\overline{DC}_{cor} - DC_{cor}^{min}} + 1\right]^{r\sin\theta}, \tag{13}$$

where the parameter θ varies in the range of $[0, 2\pi]$, and the parameter $r \in \{0, 1\}$: $r = 0$ for random connectivity, and $r = 1$ for various connectivity. Each dependency coefficient is divided by its average so that the effect of the both coefficients becomes the same on average. We then added 1 to both coefficients so that the minimum dependency coefficient among all pairs of nodes always stays 1 as a standard value independent of θ.

Fix $r = 1$, and when $\theta \in (0, \pi)$, interconnecting links become assortative; otherwise when $\theta \in (\pi, 2\pi)$, the links become disassortative. When $\theta \in (3\pi/2, \pi/2)$, high centrality nodes tend to be selected as endpoints of interconnecting links, while when $\theta \in (\pi/2, 3\pi/2)$, low centrality nodes are preferred. These variability is shown in Figure 4. However, it should be noted that there is a point we should be careful regarding the affinity between the preference toward assortativity and centrality. As Figure 5 shows, that assortative connectivity and high or low centrality pairs of endpoint nodes can coexist. Whereas, disassortative connectivity prefers to connect high and low centrality nodes. Because of this conflict between disassortative preference and centrality preference, the intra-modular connectivity would be delicate against a slight change of θ in the range of $\theta \in (\pi, 2\pi)$.

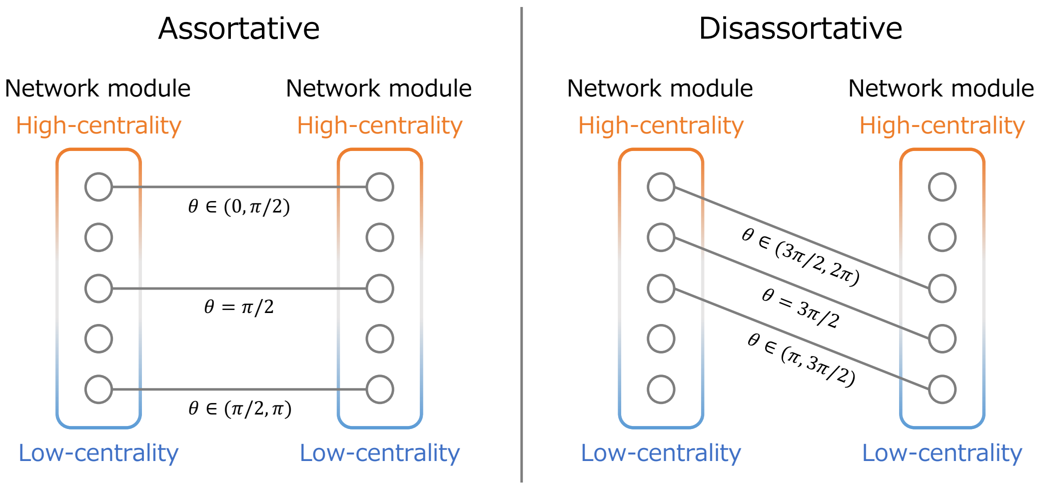


Figure 4: Various patterns of connectivity between networks

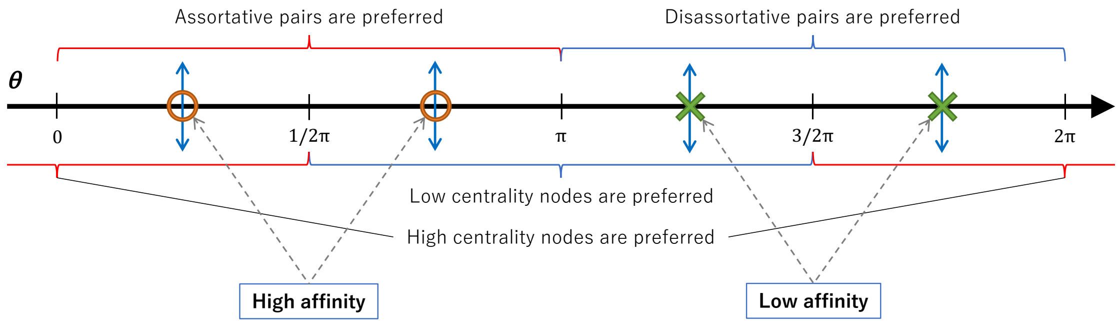


Figure 5: Relation between centrality and assortativity

5 Simulation Evaluation

In this section, we evaluate the performance of NoN models and topologies. Our aim is to reveal the way to accelerate or suppress information diffusion for communication efficiency and robustness by configuring the activation rules of NoN models and the topological connectivity. Therefore, we simulate information diffusion on NoN with various settings and measure the required time to completely diffuse. The evaluation results are shown in following subsections.

5.1 Simulation Settings

We evaluate the performance of information diffusion NoN models changing their topological connectivity. We use the IN NoN model as our proposal and the Pure NoN as a basic comparison. Their behaviors are described in Section 3.

To conduct the evaluation, we configure the parameter settings on NoN models and topologies according to Table 2. First, we generate subnetwork topologies with the preferential attachment method varying the parameter γ within the range of $\gamma \in [-50, 20]$. γ is an important parameter for changing the connectivity within a module. Although it is possible to take a wide range, higher or lower values of γ do not changes the topologies with our settings, and thus we set the range as it is shown in Table 2. Besides, we set $N = 100$ for the number of nodes in a subnetworks and $m = 2$ for the number of edges. m is used in the generating process of preferential attachment: every time a new node is added to an existing network component, m links are also added at the same time. We also set $k_{in}^{max} = 25$ as the maximum nodal degree of intra-modular links, for it is common to set a finite number of nodal degrees in information networking.

Then, we generate interconnecting links between pairs of generated subnetworks. In this thesis, we focus on interconnected networks between two subnetworks. At this time, we vary the dependency on centrality and correlation of centrality to realize various patterns of interconnectivity, changing the parameter θ. We only consider as positive r the case of r = 1, since we are not interested in random topologies in this study and because the scale of $\cos\theta$ and $\sin\theta$ in Eq. (13) does not affect the sequence of DC. θ can vary from 0 to 2π and this is another important parameter to change the connectivity between modules. Here we define $E = 25$ as the number of interconnecting links, and k_{out}^{max} takes 1 or 3. This variability of k_{out}^{max} is also important for us to know whether we should let only a few nodes have many interconnecting links or the links are

Table 2: Parameter settings

Variables	**Values**	**Description**
δ	0.5	parameter for information passing probability
γ	[-50,20]	parameter for bias in preferential attachment
m	2	parameter for the number of links in preferential attachment
r	1	parameter for connectivity between networks
θ	[0,2π]	parameter for connectivity between networks
N	100	number of nodes in a subnetworks
E	25	number of inter-modular links
k_{in}^{max}	25	maximum nodal degree of intra-modular links
k_{out}^{max}	$\{1,3\}$	maximum nodal degree of inter-modular links

distributed among many nodes.

In this evaluation, we measure the required time steps for information to diffuse over the entire NoN topology in order to determine the speed of the NoN information diffusion. As we described in Section 3, the diffusion of information follows a probabilistic method, and the probability of an input node to successfully pass the information during a time-step is defined as δ. This time, the value of δ is not so much of interest to us, and we fix it as 0.5. The starting points of the diffusion are (i) the highest loaded inter-modular links, and (ii) randomly selected inter-modular links, and this setting is a key point in this evaluation. Throughout this study we assume that different service network modules are cooperating as another service over the interconnectivity. In such situations, not only actions within modules, but also those taking place between modules are the very interest point in our research. It is also natural for interconnecting links and their endpoint nodes to be a source of important information or epidemics, since they are concentring a large amount of traffic. Although in its original behavior in the IN model nodes become empty after passing their information, we designate the source inter-modular link, i.e., the source endpoint nodes, to continuously send the information. This is because the diffusion is probabilistic and it is possible for the diffusion to disappear from the network in the first few steps.

5.2 Evaluation on Basic Properties of Interconnected Networks

Before we start evaluating the information diffusion efficiency, we investigate the basic properties of subnetworks and interconnected networks, that will allow deeper understandings on evaluation of interconnected networks in the following section.

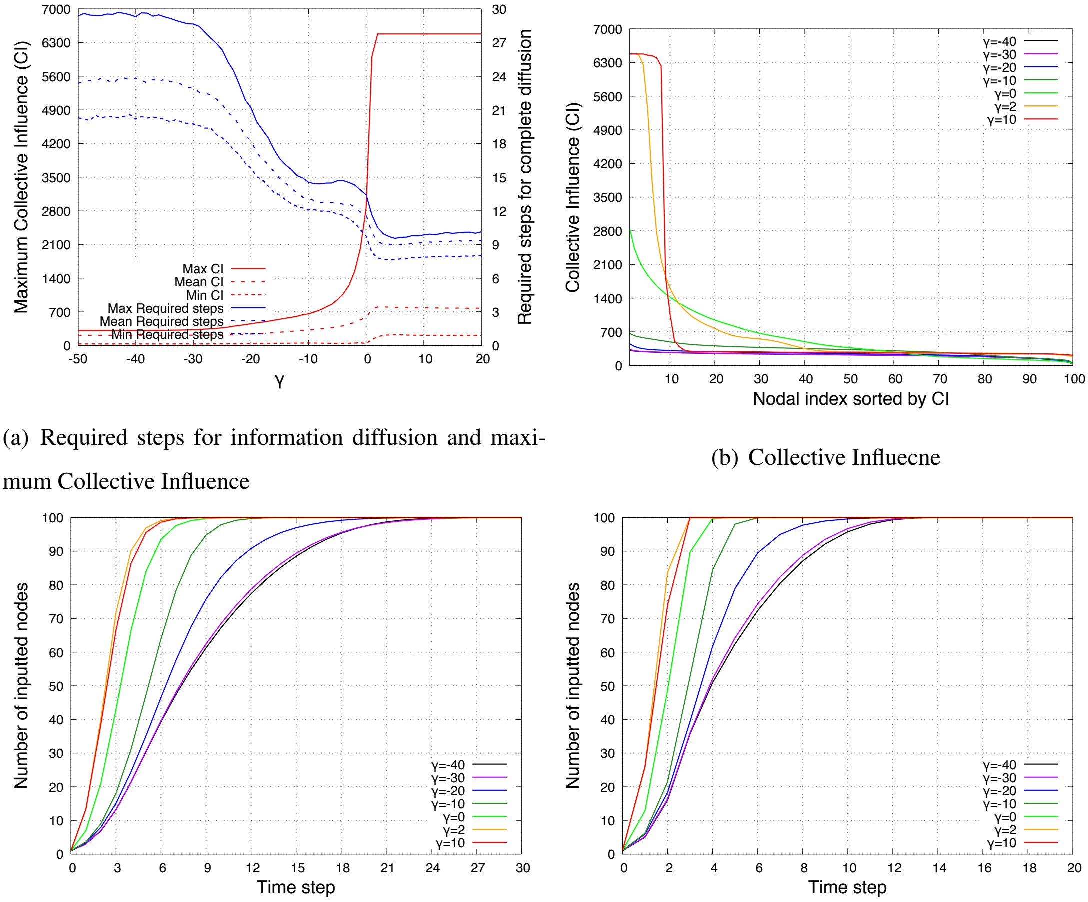
5.2.1 Subnetworks

For the verification of the method proposed in Section 4.2 that can control the bias from the existence and power of influential nodes, we generated subnetworks with $N = 100$, $m = 2$, $k_{in}^{max} = 25$, and $\gamma \in [-50, 20]$, and checked their influential nodes. We run simulations of information diffusion in these subnetworks, so that we can compare the results later when evaluating interconnected networks. Here, we set the node with maximum collective influence as the source node of the diffusion.

Figure 6(a) shows maximum collective influence and required steps for information diffusion in subnetworks. We can confirm that as the parameter γ increases, maximum collective influence successively grows and the required steps decrease. This result implies that the power of influential nodes can be summarized and distributed by changing the parameter γ. However, there is a limitation on the feasible values of maximum collective influence and required steps. Figure 6(b) shows that when $\gamma \leq -30$, the topology obtains almost completely uniform degree distribution, while when $\gamma = 10$, the nodes are separated into two groups: nodes with degrees of $k_{in}^{max} = 25$ and other peripheral nodes. Further extending the parameter γ does not lead to a change in the performances. From Figures 6(c) and 6(d) we can see a clear tendency that the speed of information diffusion increases when γ or δ increase. From those results, we can conclude that the topology construction method for subnetworks using preferential attachment we proposed in Section 4.2 certainly controls the power of influential nodes.

5.2.2 Interconnected Networks

We continue to evaluate the basic properties of interconnected networks constructed following the method described in Section 4.3. In this evaluation, we use CI as a centrality measure. In the construction of interconnected networks, we vary the value of the parameter θ and change the connectivity between networks. Figure 7 shows how the connectivity changes when θ varies. The



(a) Required steps for information diffusion and maximum Collective Influence

(b) Collective Influecne

(c) Information diffusion with $\delta = 0.5$

(d) Information diffusion with $\delta = 1.0$

Figure 6: Properties of subnetworks with various γ

red lines in the figures indicate the average centrality, i.e., collective influence, of endpoints over all interconnecting links in the interconnected network, and the blue lines indicate the average correlation of the centrality over all the interconnecting links. In other words, red lines represent the average value of DC_{cnt} in Eq. (9) and blue lines represent DC_{cor} in Eq. (12).

Although appearing complex at first sight, it turns out to follow a quite simple and fundamental rule: when $\theta = 0$ the collective influence is maximal, when $\theta = 0.5\pi$ assortativity is maximum, when $\theta = \pi$ collective influence is minimal, and when $\theta = 1.5\pi$ assortativity is minimal. This is because both centrality and correlation are ignored on these four points of θ, which is expressed in Eq. (13). On the other hand, when θ differs from these four points, a delicate interdependency between centrality and correlation arises and the values fluctuate.

Another notable point is that when γ increases, the range of centrality and correlation changes differently, and the line shapes become angular. The increase in the range of centrality is due to the emergence of influential nodes in topologies with larger γ. The shrink of the correlation range is caused by the change in nodal centrality distribution: when the centrality distribution becomes biased like the power-law shape, the feasible range of correlation becomes smaller. Besides, we can also find that topologies with $k_{out}^{max} = 1$ have narrower range of centrality and correlation. This is the result from the constraint of upper limit on nodal degree; the constraints prevents ideal interconnectivity.

5.3 Evaluation Results

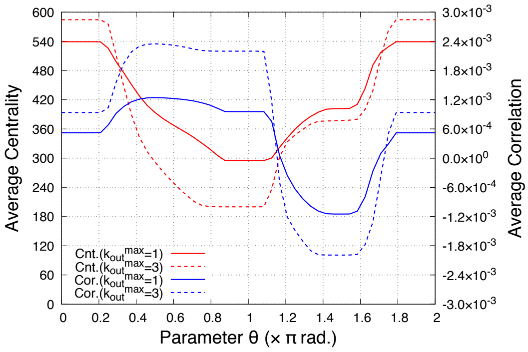
In this subsection, we investigate the performance of NoN models, IN NoN, and Pure NoN, through the simulation of information diffusion.

5.3.1 Information Diffusion Starting from Influential Sources

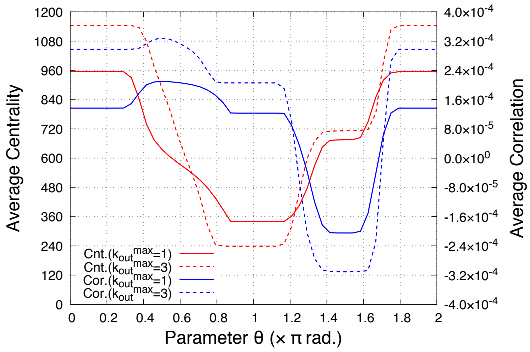
First, we simulate information diffusion selecting influential edges, i.e., an interconnecting link with influential nodes, as a source of the diffusion. We select such interconnecting links based on the average collecting influence of both endpoint nodes. An interconnecting link with the highest average is regarded as the most informative connection in the interconnected network and its endpoints play the role of source nodes of an information diffusion. The results are average of 2,000 times repetition on each parameter settings.

In Figures 8 and 9, the required time steps for information to completely diffuse all over the network is described. Solid lines are for IN NoN, and dotted lines are for Pure NoN. Shapes of the lines in Figure 8 correspond to the blue lines in Figure 6(a), which describes the information diffusion in a subnetwork. As γ increases, influential nodes gradually appear and they minimizes the diameter of each subnetwork in the interconnected network. However, the behavior of lines differ among each other, depending on the types of NoN models and the parameter θ.

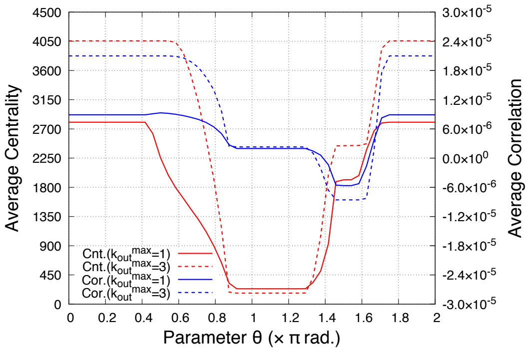
The most striking point is that solid lines of IN NoN vary more extensively than the dotted lines of Pure NoN. When γ is small, the diffusion speed greatly slows down, because each subnetwork becomes uniformly connected as we confirmed in Section 5.2.1. In such stretched networks, the endpoints of interconnecting links in each network are located far away. The activation rule for outer-interfaces of the IN NoN model requires both endpoint nodes of an interconnecting link to



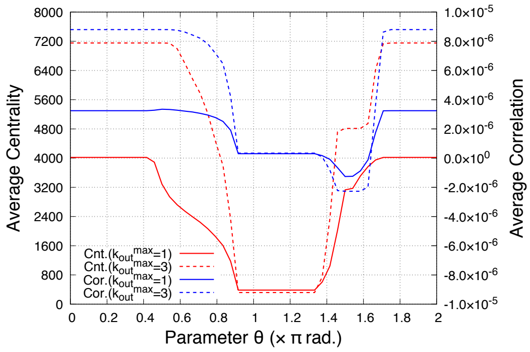
(a) $\gamma = -30$



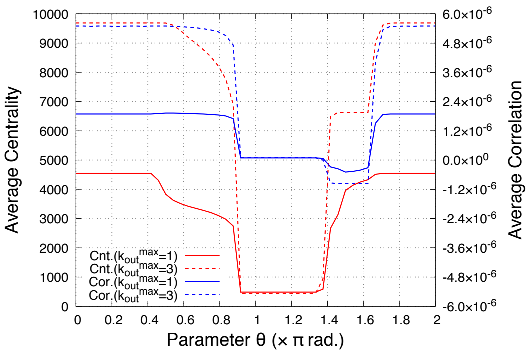
(b) $\gamma = -10$



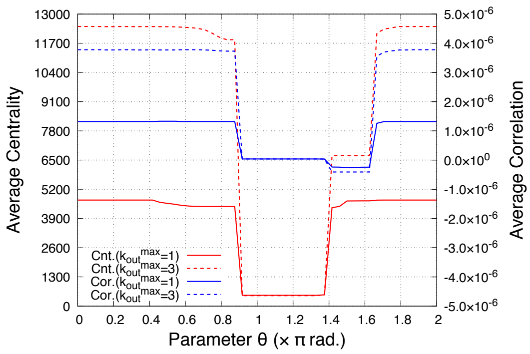
(c) $\gamma = 0$



(d) $\gamma = 1$



(e) $\gamma = 2$



(f) $\gamma = 10$

Figure 7: Centrality and correlation of interconnecting links

have input when the outer-interfaces needs to be activated. Therefore, the outer-interfaces tend to be turned off in interconnection composed of such stretched subnetworks.

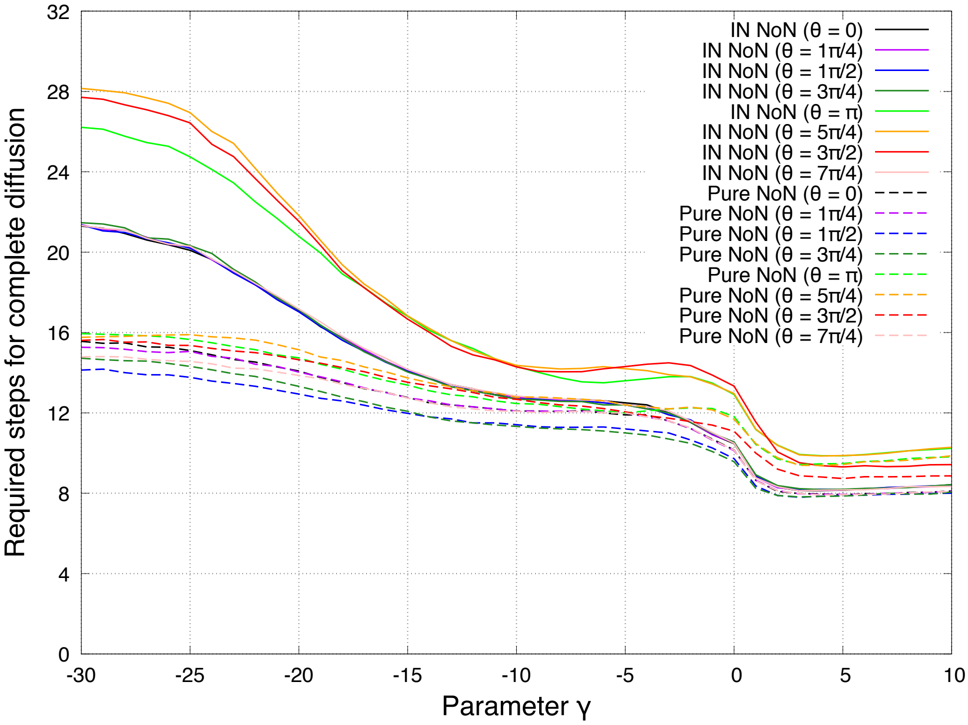
On the other hand, IN NoN achieves almost the same speed of information diffusion with Pure NoN. This nature can be seen when $\gamma \geq 2$ and $\theta \in (-0.3\pi, 0.8\pi)$, corresponding to Figure 7(f). In this range of parameters, the source interconnecting link is assumed to connect highest centrality nodes in each subnetwork according to Figure 7. Therefore, the strong diffusion sources enabled quick information diffusion for IN NoN.

When comparing $k_{out}^{max} = 1$ and $k_{out}^{max} = 3$, the diffusion speed of Pure NoN with $k_{out}^{max} = 3$ is slightly slower than that of $k_{out}^{max} = 1$. This might simply suggest that interconnecting links had better be distributed rather than concentrated on a certain set of nodes.

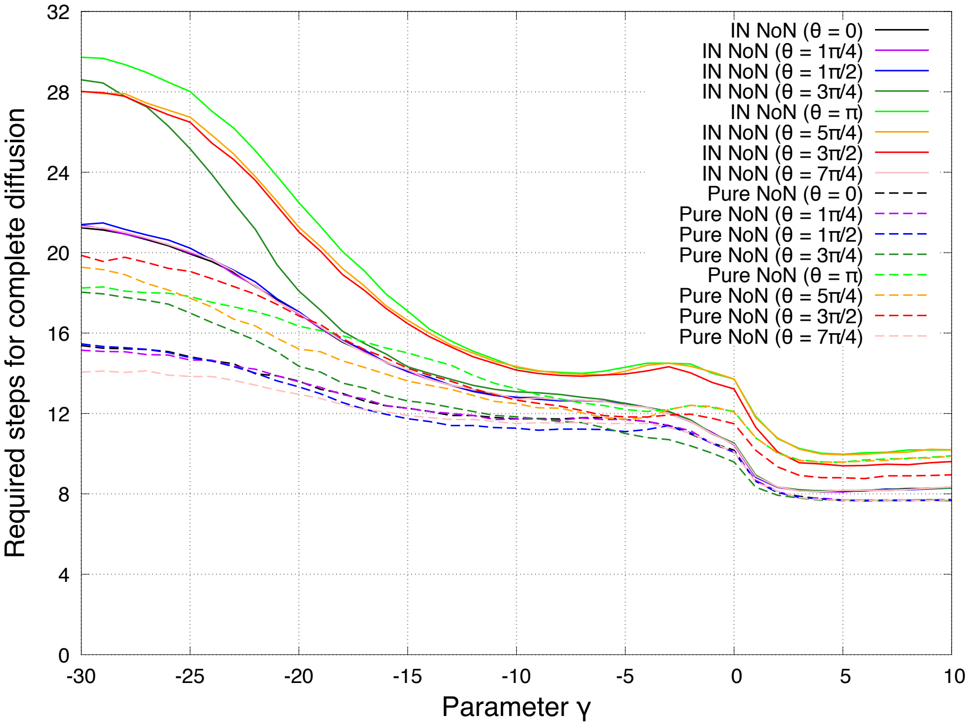
5.3.2 Information Diffusion Starting from Random Sources

Subsequently , we simulated information diffusion which starts from randomly selected interconnecting links and measured the required steps for complete diffusion. Figure 10 shows the required steps for information diffusion against the parameter γ. From this figure, we can find that the range of possible values of the required steps is slightly higher than the one in Figure 8. This is because in this simulation we choose the interconnecting links as diffusion source at random, and thus the centrality of the endpoint nodes becomes lower on average.

Another conspicuous point can be found in Fiugure 11. The results in the previous subsubsection showed the interconnected networks can reduce the diffusion time the best when $\theta \in (-0.3\pi, 0.8\pi)$ and whatever the value of γ is. However, in this case, the range of parameters that enables to minimize the diffusion speed becomes narrower as the parameter γ decreases. For example, topologies with $\gamma = 10$ can minimize the diffusion speed when $\theta \in (-0.3\pi, 0.8\pi)$, which is the same as the results in the previous subsubsection. Whereas topologies with $\gamma = -30$ can minimize the speed only within the range of $\theta \in (-0.2\pi, 0.2\pi)$. This result reflects the fact that the interconnecting links can fundamentally have endpoint nodes with high centrality around the value of $\theta = 0$. As the θ shifts farther away from $\theta = 0$, the dependency on the correlation becomes larger, and thus lower centrality nodes tend to be selected as endpoint nodes. In the simulation in the previous subsubsection, we preferentially chose interconnecting links with high centrality nodes as diffusion sources, and that made the range of θ that can minimize the diffusion speed wider.

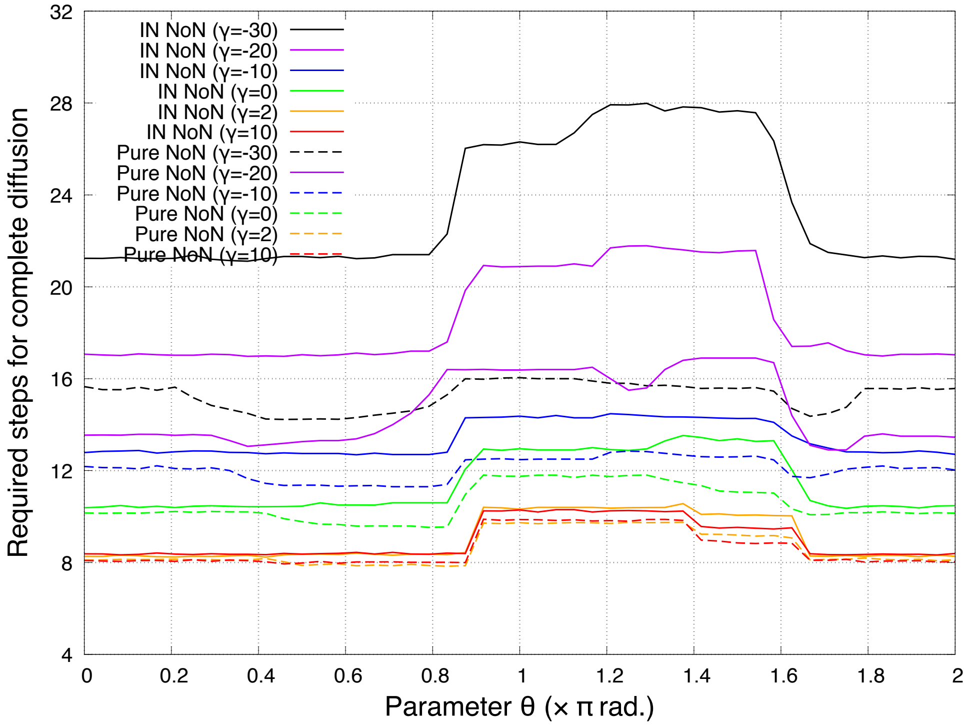


(a) $k_{out}^{max} = 1$

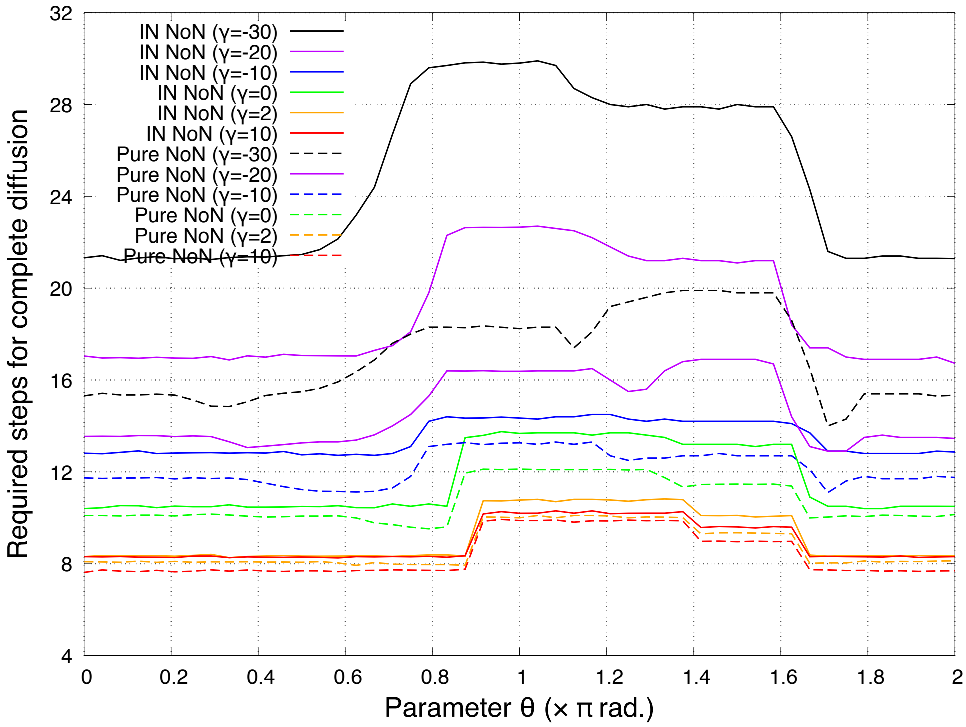


(b) $k_{out}^{max} = 3$

Figure 8: Required steps for complete diffusion with changes in connectivity within modules

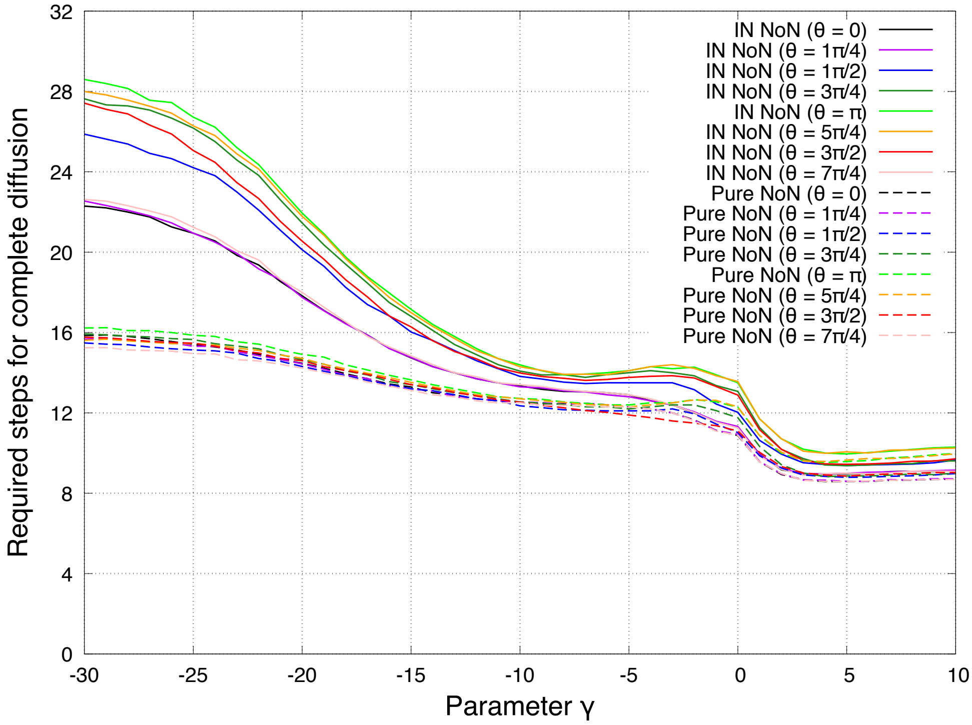


(a) $k_{out}^{max} = 1$

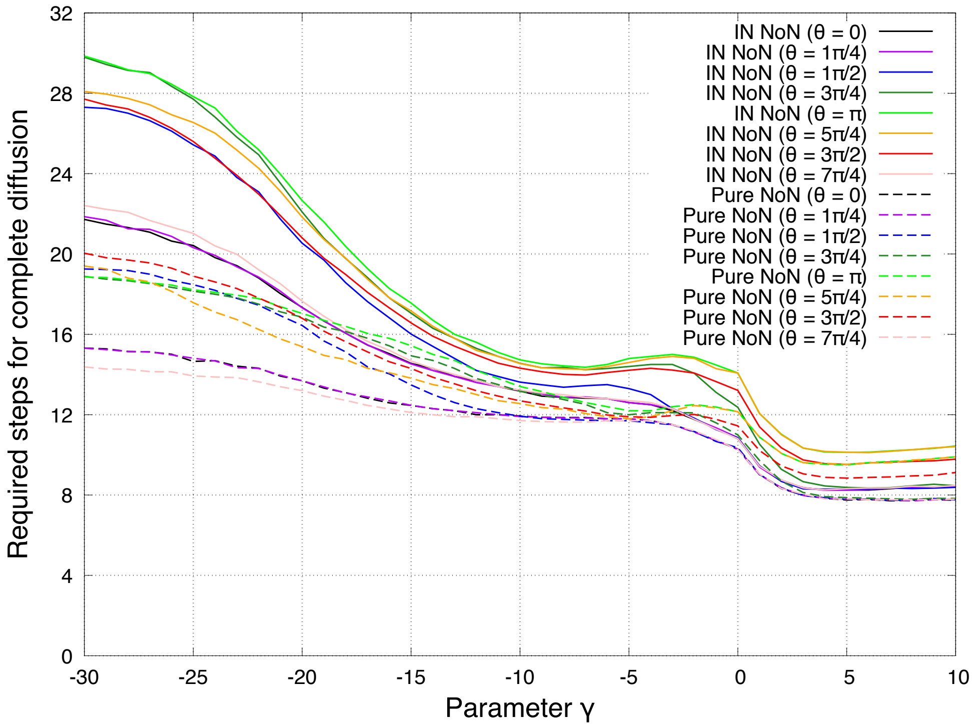


(b) $k_{out}^{max} = 3$

Figure 9: Required steps for complete diffusion with changes in connectivity between modules

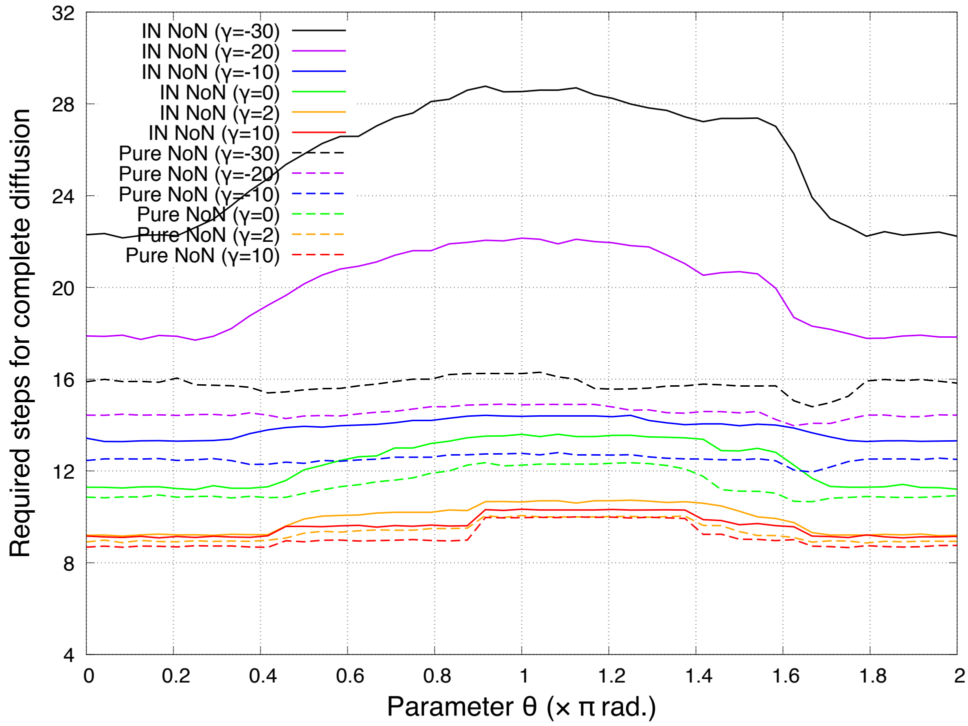


(a) $k_{out}^{max} = 1$

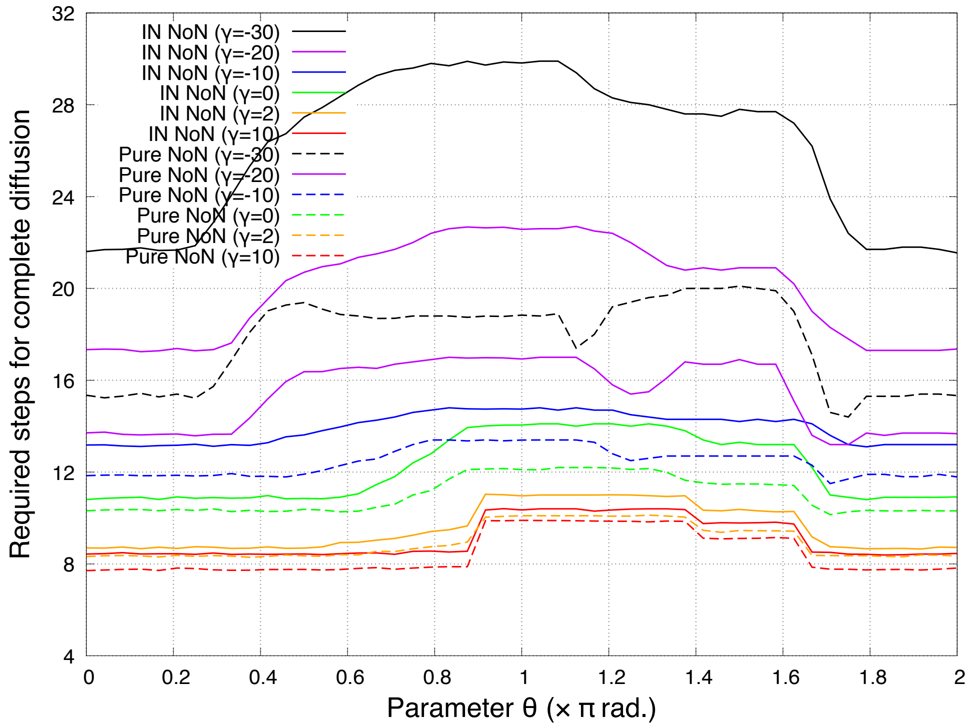


(b) $k_{out}^{max} = 3$

Figure 10: Required steps for complete diffusion with changes in connectivity within modules



(a) $k_{out}^{max} = 1$



(b) $k_{out}^{max} = 3$

Figure 11: Required steps for complete diffusion with changes in connectivity between modules

5.4 Discussion

In the activation rule in IN NoN, outer-interfaces of each node are activated only when at least one of its outer-neighbors has input. Throughout our evaluations, we confirm that this activation rule of outer-interfaces enables to modify the speed of information diffusion, in contrast to Pure NoN. The speed of information diffusion also depends on the topological connectivity within and between network modules. When we prefer to maximize the speed of information diffusion, the desired interconnected network topology can be generated by setting $\gamma > 0$ and $\theta \in (-0.2\pi, 0.2\pi)$, regardless if the diffusion starts from heavily used links or from a randomly selected ones. At this time, the term γ realizes a subnetwork with a set of extremely high centrality nodes, while the term θ generates interconnecting links between those high centrality nodes as endpoints. The speed of information diffusion could be maximized to reach almost the same as that of Pure NoN. However, when we set $\gamma < -30$ and $\theta \in (1.0\pi, 1.5\pi)$, each subnetwork becomes homogeneously stretched and interconnecting links connect nodes with low centrality and low correlation.

For the application to information networking services, the strong point of IN NoN is that it can react to situations where malicious or unacceptable information cannot path through the interconnecting links. Even if the information breaks out over the interconnecting links, configuration of connectivity within and between subnetworks can reduce the speed of the information diffusion. On the other hand, even when some important and urgent information has to travel around the interconnected networks quickly, starting the information diffusion from high centrality links can accelerate the information diffusion as fast as Pure NoN, which does not consider the prevention of malicious information and thus proposes the fastest information diffusion for the given topological connectivity. Ideally, if it is possible to manually reconfigure the connectivity for each situation, the best performance would be achieved by the IN NoN. However, It is conceivable that in real-life situations both the connectivity within and between subnetworks cannot change at the same time, depending on the specifics of the infrastructure or application services. Even in that case, our results can suggest a better way to configure the connectivity given a fixed connectivity within or between subnetworks.

In this thesis, we focused on the evaluation of information diffusion starting from interconnecting links between subnetworks. One of the reasons is that IN NoN accepts only diffusion starting from the interconnecting links. Also these links play important roles and undergo heavy processing load, and thus they are suitable as sources of information diffusion. However, we did not evaluate the performance when the diffusion starts from a different node within a subnetwork, or when interconnected networks are composed of three or more subnetworks. As we mentioned above, the current IN NoN model never transmits information from one subnetwork to another, unless the other subnetwork has independent traffic by some other sources and nodes are partially activated. Therefore, modification of IN NoN might be of interest for further simulation of information diffusion.

6 Conclusion and Future Work

In this thesis, we proposed an NoN model called IN NoN model inspired by the Brain NoN model, which reproduces the activation rule from neurons of different modules that are connected via interconnecting control links. We construct IN NoN model by reinterpreting the activation of the activation of nodes in the Brain NoN as that of interfaces. In IN NoN, we also modeled the propagation of information based on the Brain NoN. We then investigated the configuration of connectivity within and between subnetworks, so as to change the speed of information diffusion. Regarding the connectivity within modules, we configured the connectivity so that node influence over information diffusion can be accelerated or suppressed. For the connectivity between modules, we considered the properties of endpoint nodes of interconnecting links: centrality of endpoint nodes and correlation of the centrality.

As a basic characteristic, IN NoN model does not allow malicious or unaccepted information to pass through interconnecting links, and prevents the entire interconnected networks from a pandemic. However, it is conceivable that such bad information occurs on interconnecting links, since these interconnecting links play an important role on the information transmission and undergo heavy information load. Otherwise, in case of emergency when we want to pass the information quickly, we can also start the diffusion from the interconnecting links. Therefore, we simulated information diffusion starting from these interconnecting links and measured required steps for information to completely spread over interconnected networks, changing connectivity within and between subnetworks.

The results showed that IN NoN model can quickly diffuse information when we design high centrality nodes within each subnetwork and connect those nodes. At this time, the diffusion speed was as fast as Pure NoN, which does not consider the prevention of information diffusion between modules and thus proposes maximum diffusion speed with a given topology. We also found that even if malicious information spreads out from interconnecting links, we can reduce the diffusion speed to as slow as the slowest case of within the independent subnetwork, when we configure stretched subnetworks and disassortatively select non-influential nodes as endpoints of interconnecting links. That is, we could configure the connectivity so that interconnecting links never accelerate information diffusion.

In the evaluation, we focused on the information diffusion starting from interconnecting links and did not investigate the diffusion starting from a node within a subnetwork, or interconnected networks composed of three or more subnetworks. The current IN NoN model never transmits information from a subnetwork to another, unless the other subnetwork activates its nodes due to independent traffic flows. Even though preventing malicious diffusion is a strong point of IN NoN model, it would occasionally be required to pass one information to other interconnected subnetworks. Therefore, our future work would be to modify the behavior IN NoN model to match such situations.

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